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ON THE DISRUPTIVE DISCHARGE IN AIR AND LIQUID DIELECTRICS.

By T. W. Edmondson.

During the past twenty years the subject of the disruptive discharge in dielectrics and the so-called "electric strength" of dielectrics has received considerable attention. By far the greatest amount of the work done has been in connection with gaseous dielectrics and especially with air at ordinary pressures. Liquids and solids, on the other hand, have been almost entirely neglected, the only papers dealing with the potential necessary to produce a spark in such dielectrics being those of Macfarlane, Steinmetz, and Macfarlane and Pierce.

At the suggestion of Dr. Webster, the writer undertook, during the spring and summer of 1896, a series of experiments to determine the nature of the dependence of the sparking-distance upon both the electrostatic and alternating differences of potential for a number of oils such as are used for the purpose of insulating in transformers and other electrical apparatus.

At first it was hoped that direct readings of the electrostatic difference of potential could be made in the case of the oils, but this was found impracticable, and consequently a series of readings which had been made for the discharge in air were used for the purpose of reducing the indirect readings finally made. In the case of the alternating discharge the readings were direct.

History.

The first absolute measurements of the potential necessary to produce a spark-discharge in air at ordinary atmospheric pressure were made by Lord Kelvin[1] in 1860, and these were followed in more recent years by measurements made by Macfarlane,[2] Baille,[3] Bichat and Blondlot,[4] Liebig,[5] Paschen,[6] Freyberg,[7] and others. The observations of Lord Kelvin, Macfarlane, and Liebig deal with the discharge in a uniform field such as is realized in practice by the use of plane electrodes placed parallel to each other at a distance apart which is small compared with the dimensions of the plates. The remaining authors consider the phenomenon of the discharge between two spheres of equal radii, while De la Rue[8] has considered the cases in which the electrodes are two plates, two spheres of different radii, two concentric cylinders, a plane and a point, and two points, and Baille[9] and Gaugain[10] have made observations for the case of two concentric cylindrical electrodes.

The results for spheres of equal radii show remarkable agreement considering the conditions under which the experiments were performed. If the potential differences are plotted as ordinates and the spark-lengths as abscissæ, the curves obtained are found to be hyperbolic in form, the curves becoming steeper as the radii of the spheres inerease.

Acting on the suggestion of Chrystal[11] that "it would be desirable to reduce the experiments of Baille and others with spheres, in cases where the surface-density can be calculated, and to examine the values of the dielectric strength in these cases," Schuster[12] has used Kirchhoff's formula for the distribution of electricity on two electrified spheres to calculate the maximum value of F, the electric inten-

[1] Papers on Electrostatics and Magnetism, p. 247.
[2] Trans. R. S. Edin., XXVIII., p. 633, 1879.
[3] Ann. de chim. et de phys. Sér. V., XXV., p. 486, 1882.
[4] Comptes Rendus, CIII., p. 245, 1886.
[5] Phil. Mag., Series V., XXIV., p. 107, 1887.
[6] Wied. Ann., XXXVII., p. 69, 1889.
[7] Wied. Ann., XXXVIII., p. 231, 1889.
[8] Proc. R. S. Lond., XXVI., p. 519, 1877; Phil. Trans., CLXIX., p. 155, 1879.
[9] Loc. cit.
[10] Ann. de chim. et de phys., Sér. IV., VIII., p. 108, 1866.
[11] Proc. R. S. Edin., XI., p. 487, 1880–82.
[12] Phil. Mag., Series V., XXIX, p. 182, 1890.

sity at that point of the sphere having a potential V which is nearest to the sphere of zero potential; and he has in this way reduced the results of Baille and Paschen. His conclusions are as follows:

(1) For two similar systems of equal spheres in which only the linear dimensions vary, the breaking stress is greater the greater the curvature of the spheres.

(2) If the distance between the spheres is increased, the breaking stress at first diminishes.

(3) There is a certain distance for which the breaking stress is a minimum.

Using the formula obtained by Betti for the same case of the distribution of electricity, Lera[1] has reduced the results of Baille, Paschen, Freyberg, and Bichat and Blondlot, and has found the remarkable result, that, for a discharge between two equal spheres, the mean of the electric intensities calculated at the points of the spheres which are nearest to each other is constant, and that the smaller the spheres the larger will be this mean.

If F be the mean of the electric intensities calculated in this way and R the radius of the spherical electrodes, then Lera finds that

$$F = H + \frac{K}{R},$$

where H and K are constants.

The values of H and K obtained are:

(1) From Baille's results, $H = 119$, $K = 14.88$,

(2) From Paschen's results, $H = 126$, $K = 15.78$,

(3) From Freyberg's results, $H = 109$, $K = 20.33$.

As it has been stated already, the results for liquid and solid dielectrics are few in number. In two papers published in 1879, Macfarlane[2] has given measurements of the spark-length between parallel plates in oil of turpentine, paraffin oil, and olive oil, and has found that the dielectric strengths of these liquids—that is, the electric intensities which can exist in them without a discharge taking place—are sensibly constant. The measurements of potential were made

[1] Rend. della R. Accad. dei Lincei, 4th Series, VII., p. 385, 1892.

[2] Trans. R. S. Edin., XXVIII., p. 633, 1879; XXIX., p. 561, 1880.

with a Thomson reflecting electrometer and were reduced to absolute measure by comparison with a long-range absolute electrometer. In a later paper,[1] written in conjunction with Pierce, Macfarlane has devoted his attention mainly to solid dielectrics, but results are also given for kerosene. Instead of making direct readings of the potential differences, the authors compared the spark-lengths in kerosene with the equivalent spark-lengths in air, the two circuits being connected in parallel with a Toepler-Holtz machine, and the results obtained were reduced by means of the values of the air-gap expressed as differences of potential which Macfarlane had given in his previous papers. Here also the dielectric strength was found to be constant. In both series of experiments the source of electricity was a Holtz machine and the electrodes between which the spark passed were brass plates, 10 cm. in diameter, which were kept parallel to each other.

Steinmetz,[2] on the other hand, used an alternating E. M. F. of 50 volts, and his higher differences of potential, ranging to about 22,000 volts, were obtained by means of transformers. His values of maximum differences of potential in the case of turpentine, which is the only liquid common to his experiments and those of Macfarlane, are uniformly less than those obtained by the latter in the ratio of about 5 to 7. Pierce[3] attributes this difference, which is noticeable also in the case of Steinmetz's observations on air, to the fact that he separated his electrodes by pieces of glass or mica of measured thickness. This view appears to be supported by Pierce's experiments, which show that the spark passes at a lower potential when the continuity of the dielectric separating the plates is broken by pieces of solid dielectrics such as glass, than when no such break occurs. Macfarlane also has pointed out that the two sets of experiments are hardly comparable because in his own experiments the electric field was practically uniform, while in those of Steinmetz the electrodes were discs of only 5 cm. diameter, and the greatest distances to which they were separated were 2 cm. in the case of air and about .9 cm. in the case of some of the oils used. In con-

[1] Phys. Rev., I., p. 161, 1894.
[2] Trans. Amer. Inst. Elec. Eng., X., p. 84, 1893.
[3] Phys. Rev., II., p. 99, 1894.

sequence of this, the field was far from uniform, and indeed it would be a matter of extreme difficulty to calculate theoretically the electric intensity at any point of such a field.

Apparatus.

In making the absolute measurements of potential difference a guard-ring electrometer was used (Fig. 1.). The plates and the

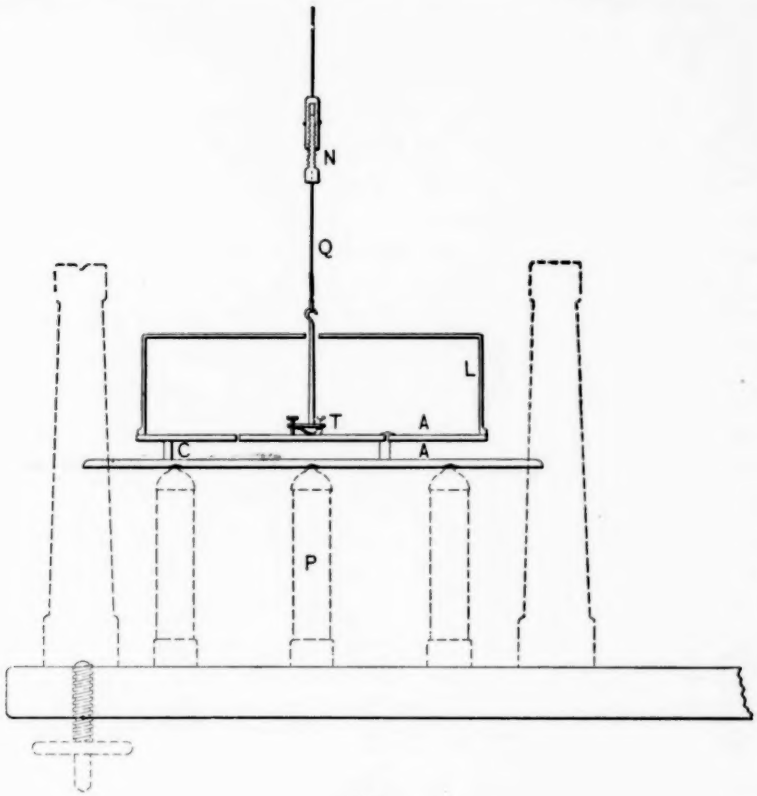

Fig. 1.

guard-ring (*A*, *A*) were made of brass, the diameter of the lower plate being about 22.5 cm., that of the movable upper plate 7.524 cm. and the inner and outer diameters of the guard-ring 7.624 cm. and about 17 cm. respectively. The lower plate was insulated by means of three hard rubber pillars *P* capped with brass points and fitted into brass sockets in a cast-iron base which served also to sup-

port the balance case. The guard-ring was kept separated from the lower plate by means of three glass cylinders C cut from a piece of plain parallel glass, 1.001 cm. in thickness; and the mean width of the slit between the guard-ring and the upper plate was 0.5 mm. The upper plate was suspended by a quartz fiber Q, of sufficient thickness to bear its weight from a screw N, which worked in a nut at the end of a copper wire which was attached to the beam of a sensitive chemical balance. The lower surfaces of the upper plate and the guard-ring, and the upper surface of the lower plate, were ground on a sheet of plate glass with fine emery and were finally polished with rotten-stone until they formed tolerably good mirrors.

After the adjustment of the lower plate for level, the guard-ring was placed in position, and the adjustment of the upper plate to the plane of the guard-ring was then made by means of the screw on the suspension wire. Its adjustment for level was made by means of a three-legged spring attached to the center of the upper surface of the plate and to the lower face of a small triangular plate T with adjusting screws at its corners. Into the center of this triangle was screwed a thin brass rod by which the plate was suspended from the end of the quartz fiber. The upper face of the movable plate was shielded from electrical influence by a light circular cover L of brass which fitted the guard-ring at its circumference.

In practice, the lower plate was grounded, and the upper plate and one electrode of the spark micrometer were connected in parallel with the inner coating of a Leyden jar, which in turn was connected with one pole of a Wimshurst machine. The other electrode, the outer coating of the jar, and the other pole of the machine were grounded. The Leyden jar served to steady the rise of potential.

The balance was loaded with weights of 0.5, 1, 1.5, 2, up to 7 grams in succession, and the beam was so checked that, when loaded, the pointer pointed to the first division of the scale to the right of the zero. At the same time, the upper plate of the electrometer was in adjustment with the plane of the guard-ring. The handle of the Wimshurst machine was then turned as uniformly as possible, and, at the same time, the distance between the electrodes was so adjusted that, at the passage of a spark, the pointer of the balance would just move to the left. A set of five readings on the

micrometer screwhead, which was divided into 500 divisions each corresponding to a distance of .001 mm. between the electrodes, was taken for each load of the balance, and the mean of these was taken as the spark length corresponding to the difference of potential as given by the weight in the scale-pan.

The closeness of the readings of the spark-micrometer is shown by the following series, by which the results given in the third column of Table II. were obtained:

Load in scale pan. grammes.	Equiv. diff. of potential. C. G. S. units.	Spark-length. mm.
.5	16.56	1.002
		.991
		1.003
		1.002
		1.002
		Mean 1.000
1.0	23.42	1.704
		1.701
		1.682
		1.678
		1.682
		Mean 1.689
1.5	28.69	2.198
		2.203
		2.205
		2.184
		2.191
		Mean 2.196
2.0	33.12	2.527
		2.513
		2.515
		2.527
		2.513
		Mean 2.519

Electrostatic Discharge.—Results.

The readings of the attraction between the plates of the electrometer were reduced to differences of potential in C. G. S. units by means of the formula

$$Wg = \frac{V^2 A}{8\pi D^2},$$

where A is Maxwell's corrected area of the plate and is equal to

$$\frac{1}{2}\pi\left\{ R^2 + R'^2 - (R'^2 - R^2)\frac{a}{D+a} \right\},$$

W is the load of the balance in grams, and $a = .22064\,(R' - R)$. The relation $V = 23.42\sqrt{W}$ was in this way obtained.

The following tables contain the results of five series of observations on sparking-distances in air between spheres of 1 cm. diameter, and of two series each for spheres of .5 cm., 2 cm., 3 cm. diameter respectively :

TABLE I.

DIELECTRIC: AIR. *Electrodes: Spheres of .5 cm. diameter.*

Difference of Potential in C.G.S. units.	Spark-length in mm.		
	Series I.	Series II.	Mean.
16.56	.958	.934	.95
23.42	1.544	1.486	1.52
28.69	2.093	2.025	2.06
33.12	2.569	2.447	2.51
37.03	2.972	2.835	2.90
40.56	3.146	3.225	3.19
43.82	3.520	3.571	3.55
46.84	3.888	3.954	3.91
49.68	4.250	4.222	4.24
52.37	4.446	4.575	4.51

TABLE II.

DIELECTRIC: AIR. *Electrodes: Spheres of 1 cm. diameter.*

Difference of Potential in C. G. S. units.	Spark-length in mm.					
	Series I.	Series II.	Series III.	Series IV.	Series V.	Mean.
16.56	.992	1.000	1.000	.993	1.019	1.00
23.42	1.649	1.689	1.592	1.634	1.560	1.62
28.69	2.054	2.196	2.062	2.176	2.067	2.11
33.12	2.418	2.519	2.487	2.505	2.484	2.48
37.03	2.892	2.872	2.883	2.976	2.892	2.90
40.56	3.193	3.239	3.238	3.326	3.200	3.24
43.82	3.491	3.479	3.492	3.626	3.486	3.51
46.84	3.827	3.855	3.846	3.936	3.860	3.86
49.68	4.114	4.201	4.156	4.102	4.089	4.13
52.37	4.386	4.439	4.371	4.360	4.345	4.38
54.92	4.599	4.591	4.574			4.59
57.37	4.895	4.942	4.879			4.90
59.71	5.149	5.158				5.15
61.96	5.334	5.313				5.32

TABLE III.

DIELECTRIC: AIR. *Electrodes: Spheres of* 2 *cm. diameter.*

Difference of Potential in C. G. S. units.	Spark-length in mm.		
	Series I.	Series II.	Mean.
16.56	1.013	1.128	1.07
23.42	1.543	1.620	1.58
28.69	2.071	2.200	2.14
33.12	2.478	2.560	2.52
37.03	2.861	2.901	2.88
40.56	3.272	3.252	3.26
46.84	3.792	3.800	3.80
52.37	4.289	4.272	4.28

TABLE IV.

DIELECTRIC: AIR. *Electrodes: Spheres of* 3 *cm. diameter.*

Difference of Potential in C. G. S. units.	Spark-length in mm.		
	Series I.	Series II.	Mean.
16.56	1.132	1.037	1.08
23.42	1.692	1.631	1.66
28.69	2.246	2.301	2.27
33.12	2.618	2.623	2.62
37.03	2.966	2.869	2.92
40.56	3.266	3.259	3.26
46.84	3.742	3.721	3.73
52.37	4.202	4.106	4.15

It will be seen, from curve I, Fig. 2, that, if the differences of potential are plotted as ordinates and the spark-lengths as abscissæ, the curves representing the results contained in Tables I.–IV. are hyperbolic in form. Using the method of least squares for the reduction of the results, it was found that the difference of potential V could be expressed very accurately in terms of d, the spark-length, by means of an equation of the form

$$V^2 = ad + bd^2,$$

where V is given in C.G.S. units of potential and d in millimetres.

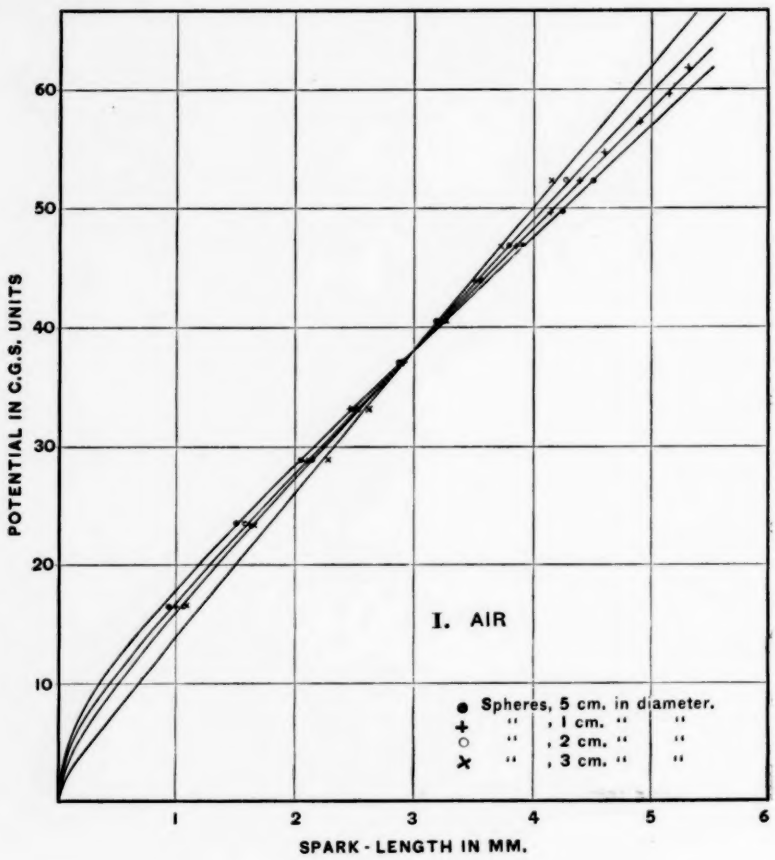

Fig. 2

The following values of a and b were obtained:

Diam. of Spheres.	a.	b.
.5cm.	235.13	83.25
1cm.	186.35	99.42
2cm.	144.41	114.49
3cm.	49.41	144.71

The degree of accuracy with which these formulæ agree with the observed results is shown by the following tables:

TABLE IA.

Electrodes: Spheres of .5cm. diameter.

d	V obs.	V calc.	$V_o - V_c$	d	V obs.	V calc.	$V_o - V_c$
mm.				mm.			
.95	16.56	17.28	—.72	3.19	40.56	39.97	+.59
1.52	23.42	23.44	—.02	3.55	43.82	43.40	+.42
2.06	28.69	28.99	—.30	3.91	46.84	46.82	+.02
2.51	33.12	33.39	—.27	4.24	49.68	49.94	—.26
2.90	37.03	37.18	—.15	4.51	52.37	52.48	—.11

TABLE IIA.

Electrodes: Spheres of 1cm. diameter.

d	V obs.	V calc.	$V_o - V_c$	d	V obs.	V calc.	$V_o - V_c$
mm.				mm.			
1.00	16.56	16.92	—.36	3.86	46.84	46.99	—.15
1.62	23.42	23.75	—.33	4.13	49.68	49.74	—.06
2.11	28.69	28.95	—.26	4.38	52.37	52.28	+.09
2.48	33.12	32.82	+.30	4.59	54.92	54.41	+.51
2.90	37.03	37.16	—.13	4.90	57.37	57.56	—.19
3.24	40.56	40.66	—.10	5.15	59.71	60.08	—.37
3.51	43.82	43.42	+.40	5.32	61.96	61.81	+.15

TABLE IIIA.

Electrodes: Spheres of 2cm. diameter.

d	V obs.	V calc.	$V_o - V_c$	d	V obs.	V calc.	$V_o - V_c$
mm.				mm.			
1.07	16.56	16.90	—.34	2.88	37.03	36.95	+.08
1.58	23.42	22.67	—.25	3.26	40.56	41.08	—.52
2.14	28.69	28.87	—.18	3.80	46.84	46.93	—.09
2.52	33.12	33.03	+.09	4.28	52.37	52.11	+.26

TABLE IVA.

Electrodes: Spheres of 3cm. diameter.

d	V obs.	V calc.	$V_o - V_c$	d	V obs.	V calc.	$V_o - V_c$
mm.				mm.			
1.08	16.56	14.91	+1.65	2.92	37.03	37.12	—.09
1.66	23.42	21.93	+1.49	3.26	40.56	41.22	—.66
2.27	28.69	29.29	— .60	3.73	46.84	46.88	—.04
2.62	33.12	33.51	— .39	4.15	52.37	51.94	+.43

The results given in Tables I.–IV. agree very well with those obtained by Baille, Bichat and Blondlot, Paschen and Freyberg, as will be seen from the following tables in which are given the values of V calculated from the formulæ on page for the spark-lengths used by the above observers.

TABLE IB.

Comparison of results with those of Paschen and Freyberg for Spheres of .5cm. diameter.

d	*V calc.*	Paschen	Freyberg	*d*	*V calc.*	Paschen	Freyberg
mm.				mm.			
1.0	17.84	16.45	16.8	3.5	42.93	43.28	
1.5	23.24	22.59		4.0	47.67	47.64	45.2
2.0	28.34	28.18	28.6	4.5	52.38	51.56	
2.5	33.29	33.60		5.0	57.07	54.67	50.5
3.0	38.14	38.65	37.2	5.5	61.74	57.27	

TABLE IIB.

Comparison of results with those of Baille, Bichat and Blondlot, Paschen and Freyberg for Spheres of 1cm. diameter.

d	*V calc.*	Baille	Bichat and Blondlot	Paschen	Freyberg
mm.					
.5	10.86	9.18			
1.0	16.93	15.25	16.1	16.11	15.5
1.5	22.43	21.28		22.17	
2.0	27.75	26.78	27.5	27.87	31.7
2.5	32.97	31.10		33.42	
3.0	38.13	37.32	38.2	39.00	38.9
3.5	43.25	42.48		44.32	
4.0	48.33	47.62	47.7	49.31	46.6
4.5	53.40	51.56		54.18	
5.0	58.46	54.66	56.3	59.03	56.0
5.5	63.53			63.35	

TABLE IIIB.

Comparison of results with those of Paschen and Freyberg for Spheres of 2cm. diameter.

d	*V calc.*	Paschen	Freyberg	*d*	*V calc.*	Paschen	Freyberg
mm.				mm.			
1.0	16.09	15.96	15.2	3.5	43.68	43.93	
1.5	21.78	21.94		4.0	49.09	49.17	48.0
2.0	27.33	27.59	29.0	4.5	54.48	54.37	
2.5	32.81	32.96		5.0	59.87	59.71	56.8
3.0	38.26	38.59	38.5	5.5	65.25	64.60	

TABLE IVB.

Comparison of results with those of Baille for Spheres of 3cm. diameter.

d	*V calc.*	Baille	*d*	*V calc.*	Baille
mm.			mm.		
.5	7.80	9.18	3.0	38.09	36.59
1.0	13.93	14.99	3.5	44.11	41.47
1.5	19.99	20.47	4.0	50.13	46.77
2.0	26.03	25.95	4.5	56.15	51.60
2.5	32.06	31.33	5.0	62.17	54.96

In making the experiments with the insulating oils it was found that it was not possible to make a consistent set of measurements of the potential by the direct method employed in the case of air. This was due to the fact that the number of sparks that had to be passed through the liquid before a satisfactory throw of the pointer of the balance was obtained, was so large that the electrodes became coated with a deposit of carbon, denser in some cases than others, and a chain of carbon particles was formed between the electrodes, thus materially altering the conditions of the discharge. This phenomenon was first noticed by Macfarlane, who calls attention also to the fact that in some cases the discharge took place by means of bubbles of gas. This mode of discharge was not observed in the experiments described in this paper, as, on account of the horizontal position of the spheres relative to each other, the bubbles of gas liberated at the passage of the spark could easily escape from the field.

Instead, therefore, of making direct measurements of the potential, the indirect method used by Macfarlane and Pierce was adopted, and, since the measurements of the spark-length in air in the case of the spheres of 1 cm. in diameter agreed very well with those of previous observers, these spheres were used as a subsidiary electrometer.

In the experiments two spark-micrometers were connected in parallel with the Wimshurst machine, one electrode of each and one pole of the machine being led to earth. One of the micrometers was placed in the liquid to be tested and its spheres were separated to a fixed distance which could be read off on the screw-head. The air-gap micrometer, which carried the spheres of 1 cm. diameter, was

then set with a shorter spark-gap than that which corresponded to the spark-gap in the liquid and the spheres were separated until the spark preferred to pass through the liquid. In Macfarlane and Pierce's experiments with solids the first spark was taken, but in the case of oils it was found that the charging of the spheres caused the attraction of small solid particles to them. These particles, in spite of the care exercised in filtering the lighter oils, it seemed impossible to get rid of. They formed a chain between the electrodes and, in consequence, the potential could be raised considerably beyond the true point of discharge before the spark would pass through the oil. It was, therefore, found advisable to pass several sparks until the discharge took place indifferently at either pair of electrodes. After each set of five readings the mean of which only is given in the following tables, the electrodes were carefully freed from the fine deposit of carbon which in any case it was impossible to avoid. With high potentials the oils, especially the thinner oils like kerosene, would be thrown into violent agitation between the electrodes, the agitation extending in a very short time to the rest of the liquid. This introduced a conduction effect which, in all probability, assisted the discharge; and in consequence the higher potentials given in the tables are probably somewhat too small.

TABLE V.

DIELECTRIC: KEROSENE.

Spark-length in liquid.	Spheres .5cm. diam.		Spheres 1cm. diam.		Spheres 2cm. diam.		Spheres 3cm. diam.	
	d in air.	V	d in air.	V	d in air.	V	d in air.	V
mm.	mm.		mm.		mm.		mm.	
.1	.64	12.65	.54	11.39	.37	9.09	.27	7.59
.2	1.74	25.01	1.41	21.46	.92	15.99	.56	11.64
.3	2.62	34.21	2.35	31.42	1.72	24.79	.90	15.76
.4	3.10	39.16	2.77	35.76	2.24	30.27	1.15	18.60
.5	3.52	43.45	3.31	41.30	2.86	36.69	1.56	23.07
.6	3.83	46.61	3.69	45.18	3.26	40.79	1.97	27.44
.8	4.33	51.68	4.20	50.36	3.79	46.20	2.68	34.84
1.0	4.74	55.83	4.63	54.72	4.24	50.77	3.29	41.10

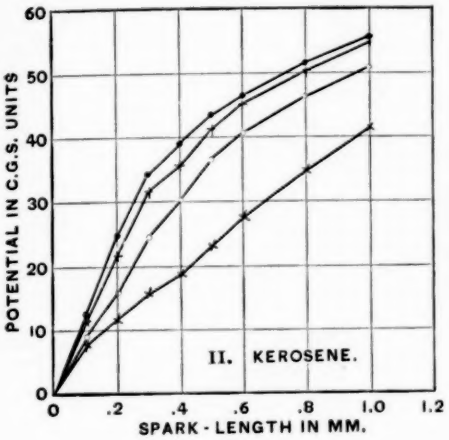

Fig. 3.

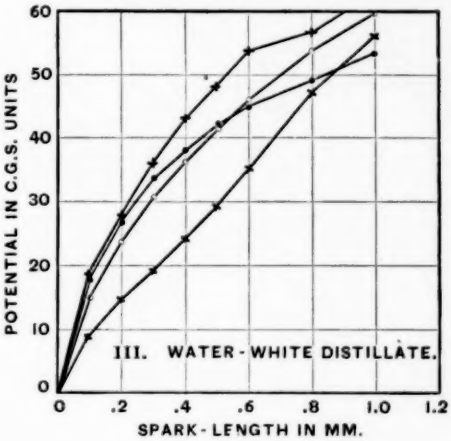

Fig. 4.

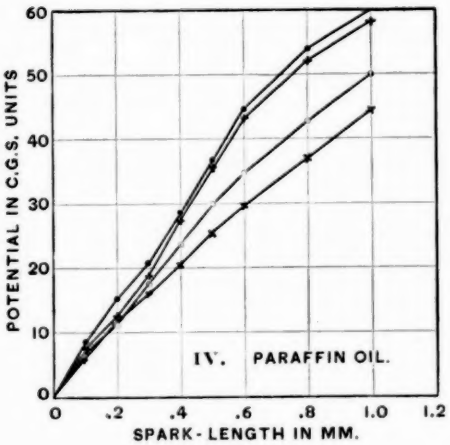

Fig. 5.

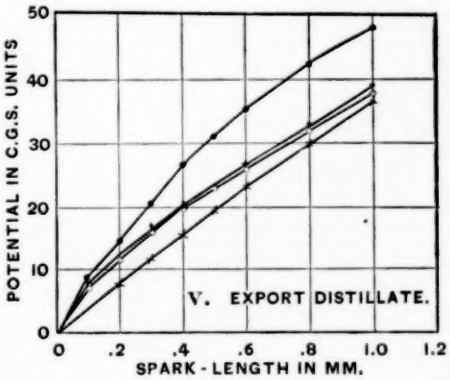

Fig. 6.

Table VI.

DIELECTRIC: WATER WHITE DISTILLATE.

Spark-length in liquid.	Spheres .5cm. diam.		Spheres 1cm. diam.		Spheres 2cm. diam.		Spheres 3cm. diam.	
	d in air.	V	d in air.	V	d in air.	V	d in air.	V
mm.	mm.		mm.		mm.		mm.	
.1	1.09	17.92	1.16	18.71	.81	14.70	.36	8.94
.2	1.91	26.81	2.00	27.76	1.61	23.62	.80	14.58
.3	2.53	33.28	2.80	36.07	2.25	30.38	1.19	19.04
.4	2.98	37.92	3.45	42.74	2.81	36.18	1.65	24.05
.5	3.36	41.82	3.99	48.23	3.31	41.30	2.13	29.12
.6	3.67	44.98	4.54	53.81	3.75	45.79	2.71	35.15
.8	4.07	49.04	4.82	56.84	4.55	53.91	3.92	47.51
1.0	4.54	53.81	5.53	63.80	5.12	59.67	4.80	56.43

Table VII.

DIELECTRIC: PARAFFIN OIL.

mm.	mm.		mm.		mm.		mm.	
.1	.30	8.05	.25	7.27	.22	6.77	.21	6.60
.2	.84	15.06	.60	12.15	.53	11.26	.54	11.39
.3	1.32	20.47	1.16	18.71	1.07	17.70	.92	15.99
.4	2.03	28.07	1.98	27.54	1.58	23.29	1.30	20.26
.5	2.83	36.38	2.76	35.66	2.19	29.75	1.75	25.11
.6	3.63	44.57	3.48	43.04	2.67	34.73	2.18	29.64
.8	4.55	53.91	4.39	52.29	3.44	42.63	2.89	37.00
1.0	5.22	60.28	4.87	58.14	4.17	50.06	3.63	44.57

Table VIII.

DIELECTRIC: EXPORT DISTILLATE.

mm.	mm.		mm.		mm.		mm.	
.1	.36	8.94	.29	7.90	.25	7.27		
.2	.79	14.47	.63	12.52	.56	11.64	.29	7.90
.3	1.33	20.58	.97	16.56	.92	15.99	.57	11.77
.4	1.86	26.28	1.29	20.15	1.26	19.82	.89	15.64
.5	2.31	31.00	1.59	23.40	1.56	23.07	1.22	19.37
.6	2.73	35.35	1.88	26.49	1.84	26.07	1.56	23.07
.8	3.41	42.33	2.42	32.15	2.41	32.05	2.22	30.06
1.0	3.97	48.02	3.06	38.74	2.97	37.82	2.84	36.59

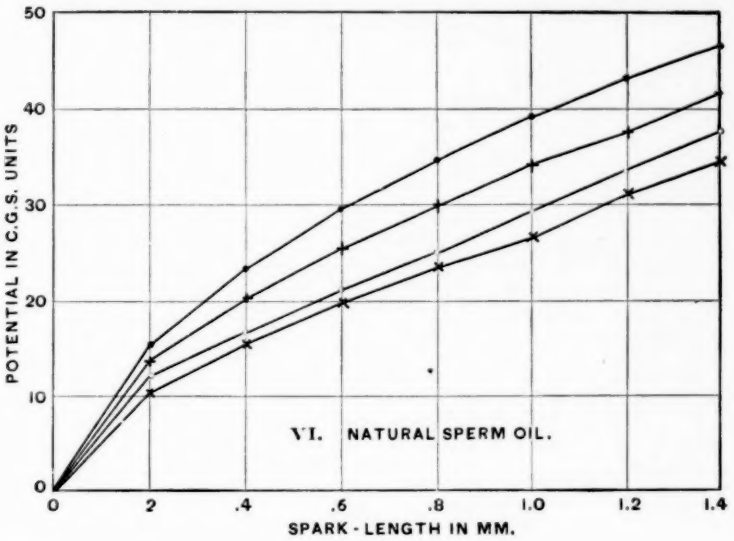
POTENTIAL IN C.G.S. UNITS
50
40
30
20
10
0
VI. NATURAL SPERM OIL.
0
2
.4
.6
.8
1.0
1.2
1.4
SPARK-LENGTH IN MM.

Fig. 7.

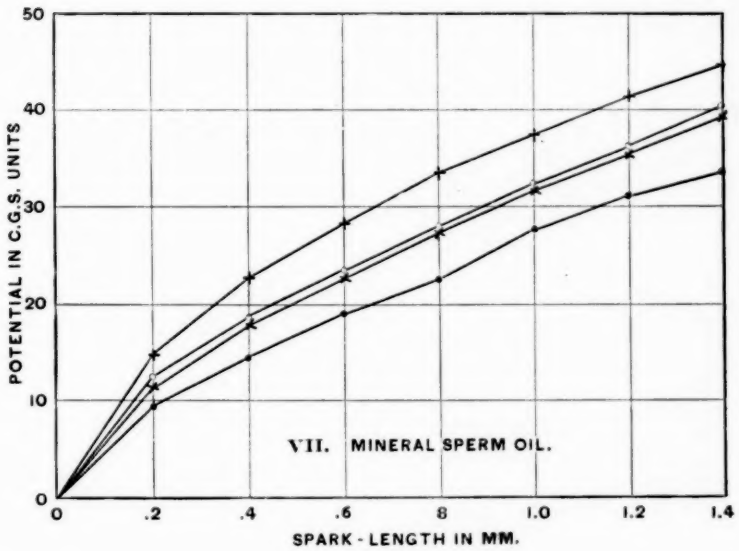
POTENTIAL IN C.G.S. UNITS
50
40
30
20
10
0
VII. MINERAL SPERM OIL.
0
.2
.4
.6
8
1.0
1.2
1.4
SPARK-LENGTH IN MM.

Fig. 8.

TABLE IX.

DIELECTRIC: NATURAL SPERM OIL.

Spark-length in liquid.	Spheres .5cm. diam.		Spheres 1cm. diam.		Spheres 2cm. diam.		Spheres 3cm. diam.	
	d in air.	V	d in air.	V	d in air.	V	d in air.	V
mm.	mm.		mm.		mm.		mm.	
.2	.86	15.29	.75	13.99	.60	12.15	.45	10.20
.4	1.57	23.18	1.30	20.26	.97	16.56	.88	15.52
.6	2.19	29.75	1.78	25.43	1.37	21.02	1.25	19.71
.8	2.67	34.73	2.21	29.96	1.74	25.01	1.57	23.18
1.0	3.10	39.16	2.52	34.21	2.14	29.23	1.86	26.28
1.2	3.48	43.04	2.94	37.51	2.55	33.49	2.31	31.00
1.4	3.83	46.61	3.35	41.72	2.93	37.41	2.65	34.53

TABLE X.

DIELECTRIC: MINERAL SPERM OIL.

mm.	mm.		mm.		mm.		mm.	
.2	.40	9.51	.81	14.70	.64	12.65	.56	11.64
.4	.79	14.47	1.51	22.54	1.14	18.48	1.07	17.70
.6	1.18	18.93	2.06	28.39	1.59	23.40	1.53	22.76
.8	1.51	22.54	2.53	33.49	2.02	27.97	1.98	27.54
1.0	2.00	27.76	2.94	37.51	2.42	32.15	2.37	31.62
1.2	2.32	31.10	3.33	41.51	2.80	36.07	2.76	35.66
1.4	2.57	33.70	3.67	44.98	3.21	40.28	3.12	39.36

TABLE XI.

DIELECTRIC: RAW LINSEED OIL.

mm.	mm.		mm.		mm.		mm.	
.2	.72	13.63	.66	12.90	.58	11.90	.51	11.00
.4	1.26	19.82	1.24	19.60	1.13	18.37	.99	16.79
.6	1.77	25.32	1.74	25.01	1.62	23.72	1.44	21.78
.8	2.30	30.90	2.22	30.06	2.10	28.81	1.83	25.96
1.0	2.80	36.07	2.69	34.94	2.52	33.18	2.22	30.06
1.2	3.18	39.98	3.09	39.05	2.90	37.10	2.61	34.11
1.4	3.56	43.86	3.48	42.81	3.22	40.38	2.97	37.82

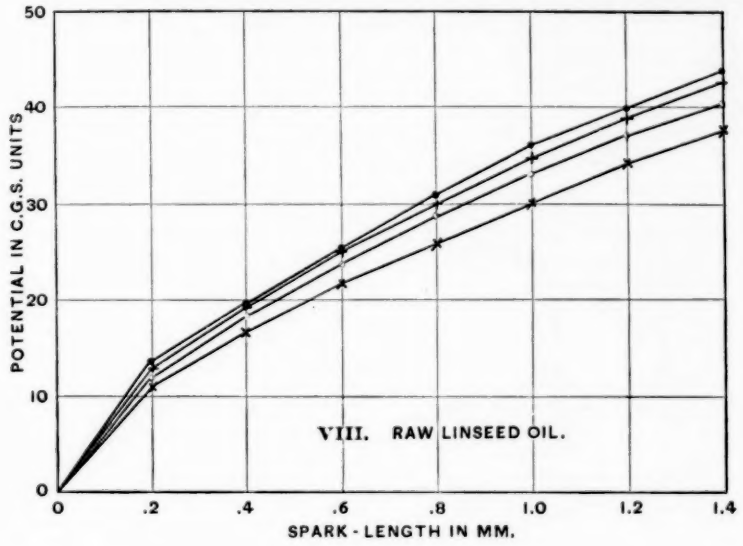
50
40
30
20
10
0
POTENTIAL IN C.G.S. UNITS
VIII. RAW LINSEED OIL.
0
.2
.4
.6
.8
1.0
1.2
1.4
SPARK - LENGTH IN MM.

Fig. 9.

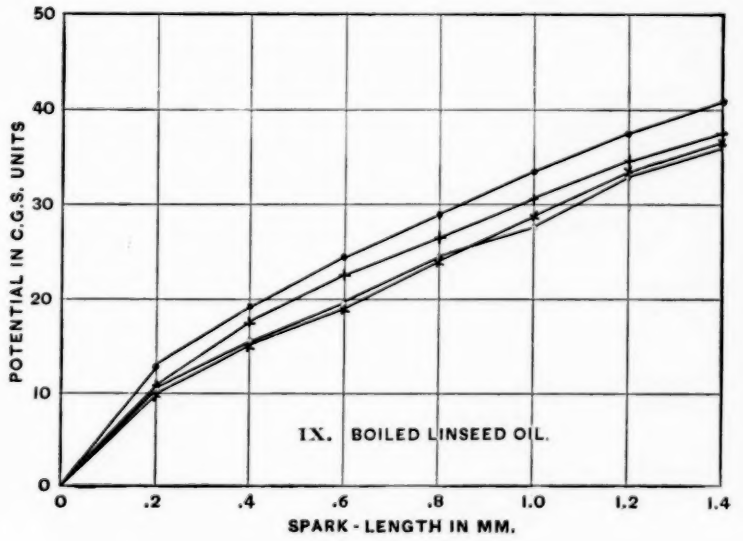
50
40
30
20
10
0
POTENTIAL IN C.G.S. UNITS
IX. BOILED LINSEED OIL.
0
.2
.4
.6
.8
1.0
1.2
1.4
SPARK - LENGTH IN MM.

Fig. 10.

TABLE XII.

DIELECTRIC: BOILED LINSEED OIL.

Spark-length in liquid.	Spheres .5cm. diam.		Spheres 1cm. diam.		Spheres 2cm. diam.		Spheres 3cm. diam.	
	d in air.	V	d in air.	V	d in air.	V	d in air.	V
mm.	mm.		mm.		mm.		mm.	
.2	.65	12.77	.50	10.86	.47	10.47	.43	9.93
.4	1.19	19.04	1.04	17.36	.88	15.52	.84	15.06
.6	1.66	24.15	1.50	22.43	1.24	19.60	1.19	19.04
.8	2.11	28.91	1.88	26.49	1.66	24.15	1.65	24.05
1.0	2.55	33.49	2.29	30.79	2.00	27.76	2.10	28.81
1.2	2.93	37.41	2.68	34.84	2.51	33.08	2.54	33.38
1.4	3.27	40.90	2.97	37.82	2.82	36.28	2.85	36.59

TABLE XIII.

DIELECTRIC: OLIVE OIL.

mm.	mm.		mm.		mm.		mm.	
.2	.76	14.11	.56	11.64	.47	10.47	.40	9.51
.4	1.47	22.11	1.07	17.70	.90	15.76	.79	14.47
.6	2.06	28.39	1.53	22.76	1.32	20.47	1.14	18.48
.8	2.62	34.21	1.88	26.49	1.59	23.40	1.45	21.89
1.0	3.12	39.36	2.31	31.00	1.91	26.81	1.74	25.01
1.2	3.58	44.06	2.70	35.04	2.11	28.91	1.97	27.44
1.4	3.98	48.13	3.01	38.23	2.34	31.31	2.19	29.75

TABLE XIV.

DIELECTRIC: NEAT'S FOOT OIL.

mm.	mm.		mm.		mm.		mm.	
.2	.90	15.76	.83	14.94	.58	11.90	.47	10.47
.4	1.82	25.86	1.57	23.18	1.04	17.36	.86	15.29
.6	2.36	31.52	2.20	29.85	1.47	22.11	1.23	19.48
.8	2.92	37.31	2.82	36.28	1.92	26.91	1.56	23.07
1.0	3.60	44.26	3.42	42.43	2.32	31.10	1.84	26.07
1.2	4.13	49.65	3.85	46.81	2.69	34.94	2.10	28.81
1.4	4.58	54.21	4.21	50.46	2.98	37.92	2.32	31.10

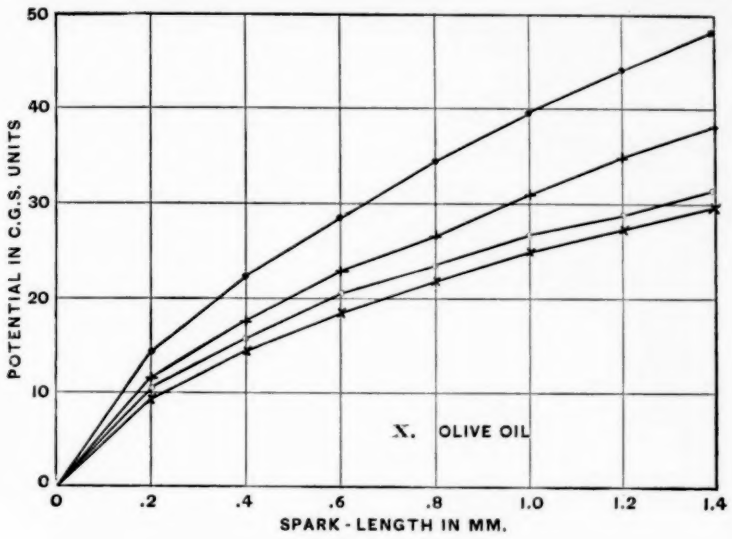
POTENTIAL IN C.G.S. UNITS
SPARK-LENGTH IN MM.
X. OLIVE OIL
0
10
20
30
40
50
.2
.4
.6
.8
1.0
1.2
1.4

Fig. 11.

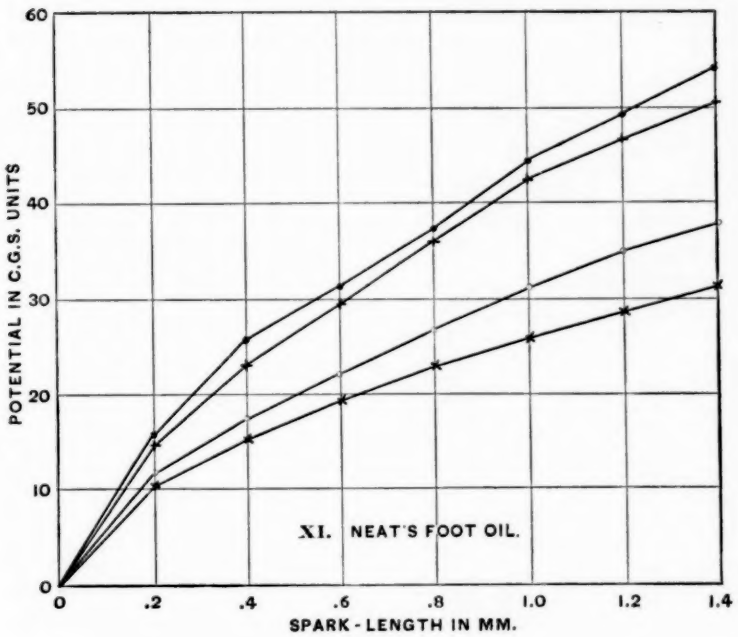
POTENTIAL IN C.G.S. UNITS
SPARK-LENGTH IN MM.
XI. NEAT'S FOOT OIL.
0
10
20
30
40
50
60
.2
.4
.6
.8
1.0
1.2
1.4

Fig. 12.

TABLE XV.

DIELECTRIC: CASTOR OIL.

Spark-length in liquid.	Spheres .5cm. diam.		Spheres 1cm. diam.		Spheres 2cm. diam.		Spheres 3cm. diam.	
	d in air.	V	d in air.	V	d in air.	V	d in air.	V
mm.	mm.		mm.		mm.		mm.	
.2	.56	11.64	.74	13.87	.69	13.26	.62	12.40
.4	1.26	19.82	1.56	23.07	1.43	21.68	1.30	20.26
.6	1.89	26.49	2.34	31.31	2.11	28.91	2.02	27.97
.8	2.52	33.18	3.08	38.95	2.89	37.00	2.68	34.84
1.0	3.18	39.97	3.82	46.50	3.56	43.86	3.29	41.10
1.2	3.85	46.81	4.53	53.71	4.21	50.46	3.88	47.12

TABLE XVI.

DIELECTRIC: LARD OIL.

mm.	mm.		mm.		mm.		mm.	
.2	.34	8.65	.77	14.23	.38	9.23	.33	8.50
.4	.62	12.40	1.30	20.26	.64	12.65	.60	12.15
.6	.85	15.17	1.59	23.40	.86	15.29	.84	15.06
.8	1.00	16.90	1.66	24.15	1.06	17.59	1.04	17.36
1.0	1.12	18.26	1.74	25.01	1.26	19.82	1.23	19.48
1.2	1.23	19.48	1.81	25.75	1.45	21.89	1.40	21.35
1.4	1.33	20.58	1.87	26.38	1.61	23.62	1.56	23.07
1.6	1.43	21.68	1.94	27.12	1.75	25.11	1.70	24.58
1.8	1.52	22.65	2.00	27.76	1.89	26.60	1.80	25.64
2.0	1.63	23.83	2.06	28.39	1.99	27.65	1.88	26.49

TABLE XVII.

DIELECTRIC: TURPENTINE.

mm.	mm.		mm.		mm.		mm.	
.2	.80	14.58	.74	13.87	.37	9.09	.27	7.59
.4	1.91	26.81	1.57	23.18	.95	16.33	.66	12.90
.6	2.68	34.84	2.44	32.35	1.53	22.76	1.07	17.70
.8	3.26	40.79	3.12	39.36	2.22	30.06	1.54	22.86
1.0	3.79	46.20	3.67	44.98	2.73	35.35	1.70	24.58
1.2	4.40	52.39	3.99	48.23	3.06	38.74	2.30	30.90

TABLE XVIII.

DIELECTRIC: XYLOL.

Spark-length in liquid.	Spheres .5cm. diam.		Spheres 1cm. diam.		Spheres 2cm. diam.		Spheres 3cm. diam.	
	d in air.	V	d in air.	V	d in air.	V	d in air.	V
mm.	mm.		mm.		mm.		mm.	
.2	1.00	16.90	.40	9.51	.29	7.90	.22	6.77
.4	1.87	26.38	.80	14.58	.52	11.13	.50	10.86
.6	2.43	32.25	1.15	18.60	.79	14.47	.80	14.58
.8	3.08	38.95	1.59	23.40	1.04	17.36	1.07	17.70
1.0	3.58	44.06	2.03	28.07	1.45	21.89	1.35	20.80
1.2	4.17	50.06	2.44	32.35	1.74	25.01	1.57	23.18

It will be seen, on examination of curve I, Figs. 13–16, in which are plotted the curves representing the results of Tables I.–IV., that, while a smaller difference of potential is necessary to produce a discharge through a given distance for large spheres than for small ones

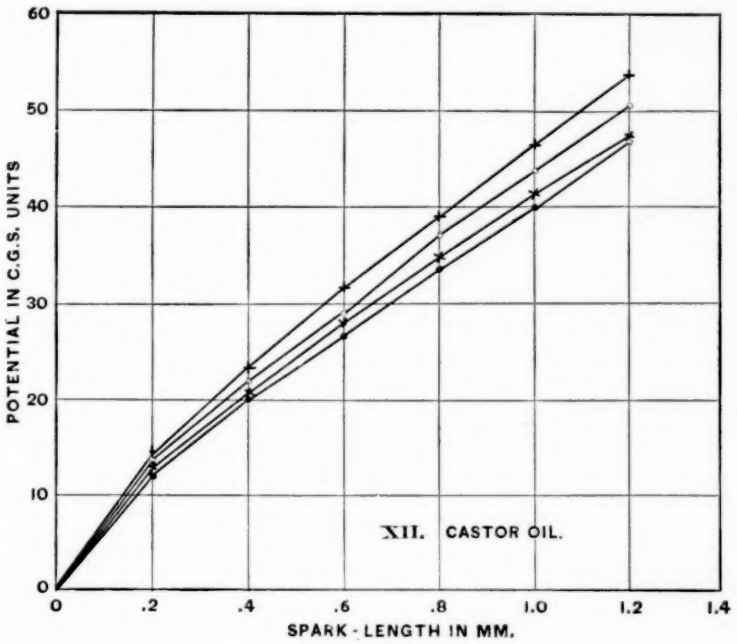

Fig. 13.

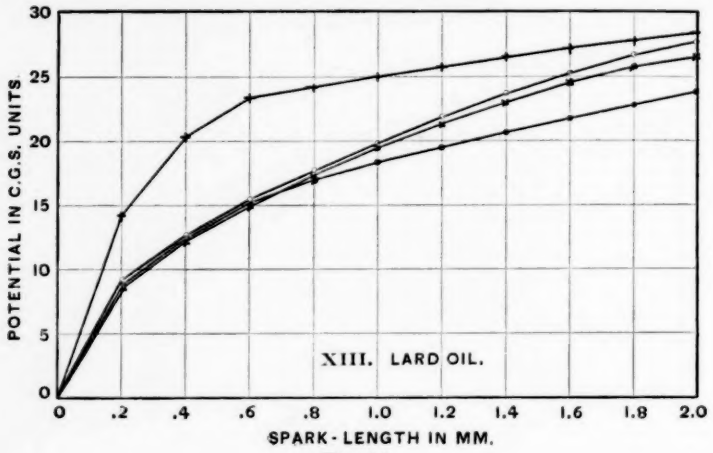

Fig. 14.

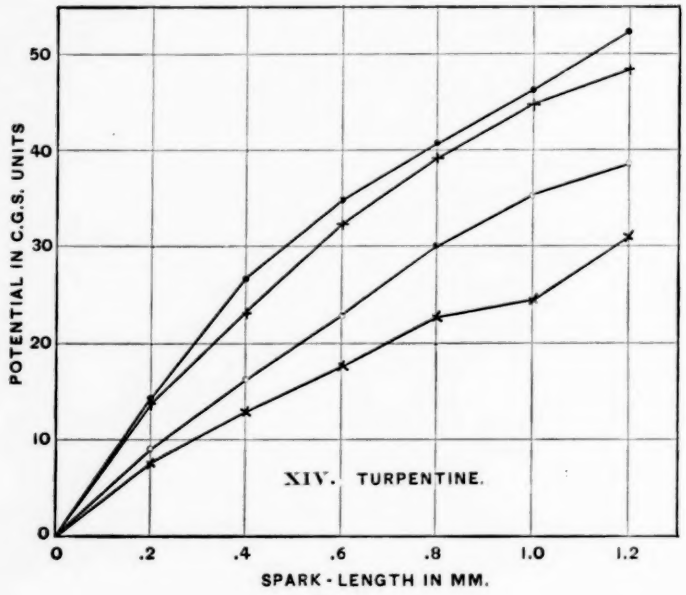

Fig. 15.

when the spheres are close together, for longer distances the dielectric is electrically stronger for large than for small spheres. In other words, although at first the increase of potential per unit length of sparking distance is greatest for small spheres and diminishes as the

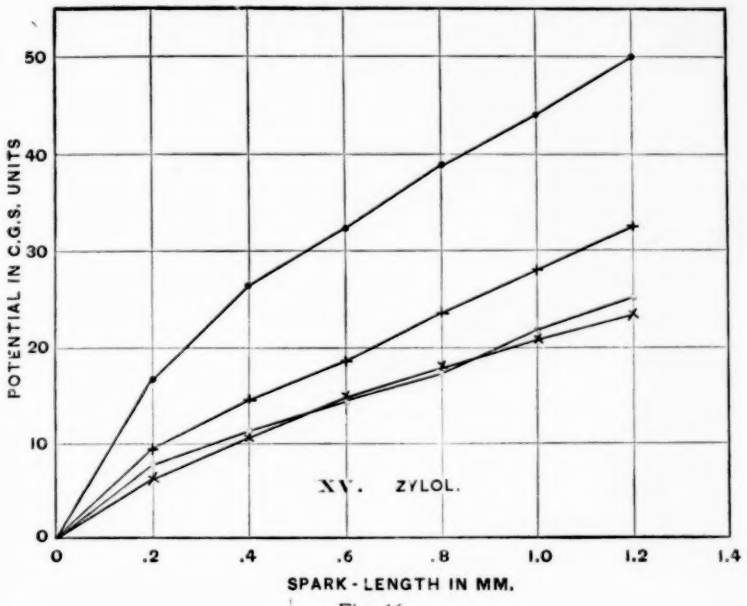

Fig. 16.

radii of the spheres increase, the rate at which this increase diminishes is most rapid for small spheres. For spark-lengths of more than 3 mm., the curves are practically straight, and the dielectric strength is therefore constant.

Taking the increase of potential while the spark-length increases from 3 mm. to 5 mm. as the basis of calculation, the values of the dielectric strength of air at ordinary pressure are as follows:

Radius of Spheres. cm.	Dielectric Strength. C. G. S. units per cm.	Kilovolts per cm.
.5	95	28.5
1.0	102	30.6
2.0	108	32.4
3.0	120	36.0

All of these values are considerably higher than that obtained by Macfarlane for planes, viz., 23.8 kilovolts per cm.

It appears then that the potential necessary for a discharge when the spheres are close together is dependent to a great extent upon the size of the spheres as well as upon the nature of the electric field.

In that case the field is almost uniform for large spheres and a larger area of the practically plane layer of dielectric is subject to a normal stress, thereby rendering the passage of the spark easier. When the spheres are so far separated that the tubes of force are appreciably curved at a short distance from the line joining the centers of the spheres, what may be considered as the normal action of the charges comes into play. The electric intensity on a small sphere is greater than on a large sphere for any given potential, and in consequence the discharge passes most easily in the case of the small spheres.

The results for the insulating oils are by no means so uniform as those for air. In the cases of water white distillate (a light oil resembling kerosene), mineral sperm oil, castor oil and lard oil the behavior of the smallest spheres is somewhat anomalous. For the other spheres the results are of the same character as those obtained for air; that is, for small spark-lengths, the dielectric is electrically strongest when the spherical electrodes are smallest. Whether the weakening of the dielectric for small spheres when the spark-length increases, takes place here also it is difficult to say, as it was impossible to obtain consistent readings for spark-lengths of more than 1.4 mm. on account of the great disturbance of the liquid due to convection currents. There appear, however, to be some indications of such a change in several cases, notably in those of kerosene, water white and export distillates, lard oil, and xylol, where the curves begin to converge.

	Dielectric Strength.		
	C. G. S. units per cm.	Kilovolts per cm.	Macfarlane.
Kerosene.	372	112	
Water White Distillate.	528	158	
Paraffin Oil.	422	127	87
Export Distillate.	319	96	
Natural Sperm Oil.	201	60	
Mineral Sperm Oil.	231	69	
Raw Linseed Oil.	223	67	
Boiled Linseed Oil.	222	67	
Olive Oil.	169	51	82
Neat's Foot Oil.	172	52	
Castor Oil.	347	104	
Lard Oil.	90	27	
Turpentine.	233	70	94
Xylol.	164	49	

The curves for spheres of 3 cm. diameter are fairly straight and it would appear that Macfarlane's conclusion that the dielectric strength of liquids is constant when the electrodes are plane is warranted. If we make use of the first and last values of potential in the last columns of Tables V.–XVIII., and assume the curve to be straight, we can obtain a lower limit to the dielectric strength of the various oils for the case of plane electrodes.

It will be noticed that, with the exception of lard oil, all the above oils are dielectrically stronger than air.

The dielectric constants of these oils have been determined by Mr. A. L. Clark, whose paper is to be found in this issue of the PHYSICAL REVIEW.

ALTERNATING DISCHARGE. RESULTS.

The results of the experiments with the alternating current were not very satisfactory, the measurements of spark-lengths showing remarkable departures from the evenness of the results obtained for the electrostatic discharge.

The source of potential was a large induction coil from which the make and break attachment had been removed. Through the primary of this coil was passed an alternating current of E. M. F. 50 volts and frequency 125, which was obtained from the ordinary street service. The two plates of the electrometers and the spark-micrometer were connected with the secondary coil in parallel, so that the induction coil served as a large open-core transformer. By means of an adjustable resistance the difference of potential could be regulated as to correspond to a given load of the balance and the sparking-distance was obtained by turning the micrometer-screw until at the passage of a spark the pointer of the balance was just thrown to the left.

In order to diminish the risk of sparking between the plates of the electrometer, glass cylinders 1.817 cm. high were used in this case to separate them, and the adjustment of the plates was made in precisely the same way as before.

For this distance between the plates, Maxwell's formula gives

$$V = 42.50\sqrt{W}.$$

But since the mean attraction only is measured by the balance the maximum difference of potential will be given by

$$V = 42.50\sqrt{2W}.$$

The results of the experiments are given in the following Tables:

TABLE XIX.

DIELECTRIC: AIR.

Maximum Potential-Difference.	Spheres .5cm. diam.	Spheres 1cm. diam.	Spheres 2cm. diam.	Spheres 3cm. diam.	Planes; calc. from Steinmetz' results.	Planes; calc. from Siemens' results.
	d	d	d	d	d	d
C. G. S. units.	mm.	mm.	mm.	mm.	mm.	mm.
26.98	2.32	1.94	2.85	3.18	4.80	2.47
38.01	3.05	3.28	4.26	4.45	7.36	3.69
46.55	4.38	5.01	4.93	5.23	9.56	4.72
53.76	5.30	6.51	5.40	6.58	11.45	5.62

TABLE XX.

DIELECTRIC: KEROSENE.

Maximum Potential-Difference.	Spheres .5cm diam.	Spheres 1cm. diam.	Spheres 2cm. diam.	Spheres. 3cm. diam.
	d	d	d	d
C. G. S. units.	mm.	mm.	mm.	mm.
26.98	.38	.29	.43	.52
38.01	.44	.54	.73	.86
46.55	.50	.75	.94	1.14
53.76	.54	.89	1.12	1.36

TABLE XXI.

DIELECTRIC: TURPENTINE.

Maximum Potential-Difference.	Spheres .5cm. diam.	Spheres 1cm. diam.	Spheres 2cm. diam.	Spheres 3cm. diam.	Planes; Calc. from Steinmetz' results.
	d	d	d	d	d
C. G. S. units.	mm.	mm.	mm.	mm.	mm.
26.98	.94	.68	.75	1.20	1.27
38.01	1.04	1.06	1.37	1.66	1.79
46.55	1.13	1.64	1.94	2.00	2.20
53.76	1.23	1.90	2.20	2.40	2.53

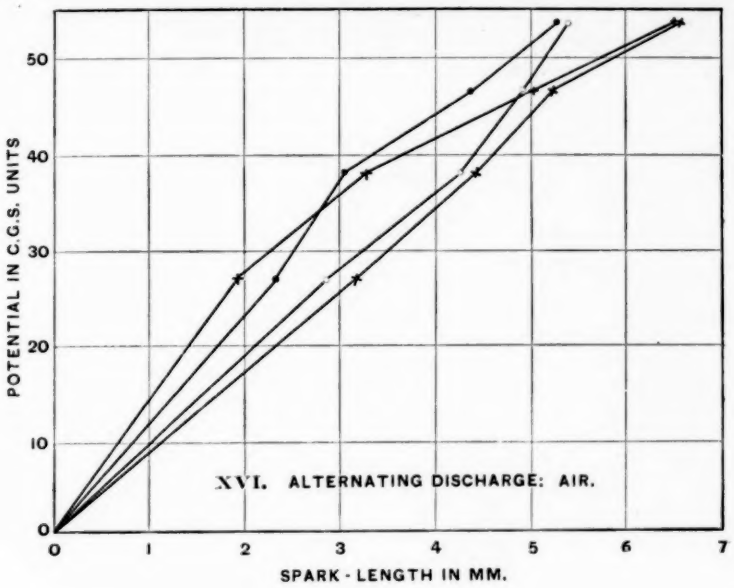
POTENTIAL IN C.G.S. UNITS
0
10
20
30
40
50
0
1
2
3
4
5
6
7
SPARK-LENGTH IN MM.
XVI. ALTERNATING DISCHARGE: AIR.

Fig. 17.

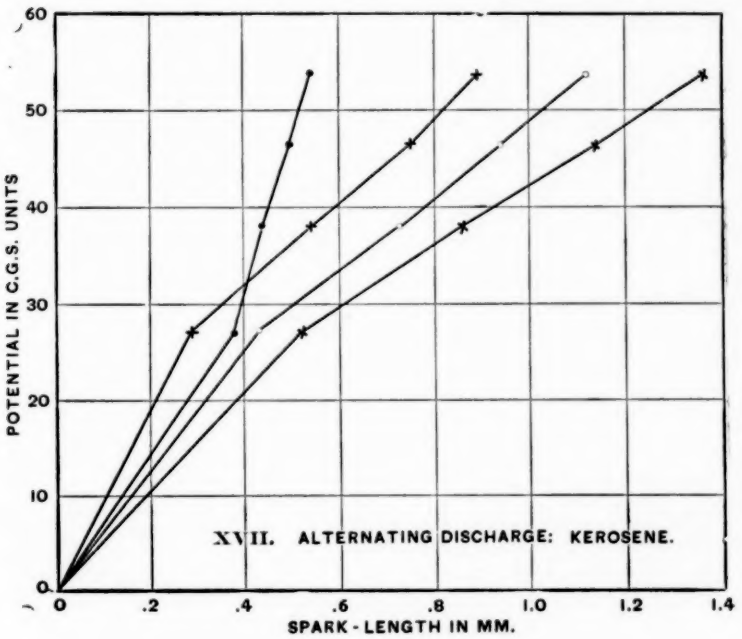
POTENTIAL IN C.G.S. UNITS
0
10
20
30
40
50
60
0
.2
.4
.6
.8
1.0
1.2
1.4
SPARK-LENGTH IN MM.
XVII. ALTERNATING DISCHARGE: KEROSENE.

Fig. 18.

TABLE XXII.

DIELECTRIC: BOILED LINSEED OIL.

Maximum Potential-Difference.	Spheres .5cm. diam.	Spheres 1cm. diam.	Spheres 2cm. diam.	Spheres 3cm, diam.	Planes; Calc. from Steinmetz' results.
	d	*d*	*d*	*d*	*d*
C. G. S. units.	mm.	mm.	mm.	mm.	mm.
26.98	.48	.55	.68	.94	1.01
38.01	.86	.87	1.06	1.11	1.43
46.55	1.00	1.22	1.34	1.38	1.75
53.76	1.34	1.42	1.56	1.66	2.01

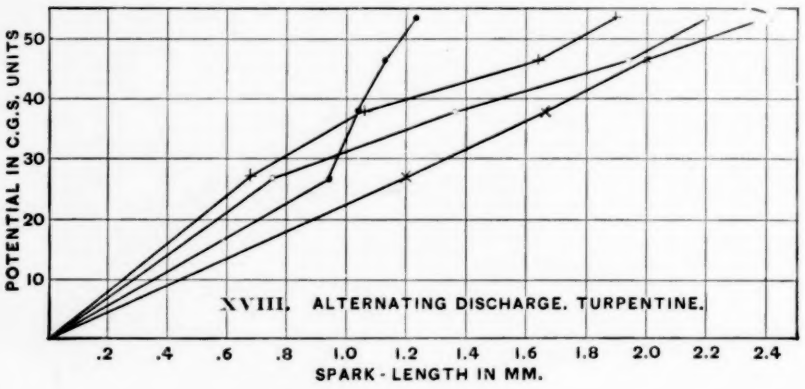

Fig. 19.

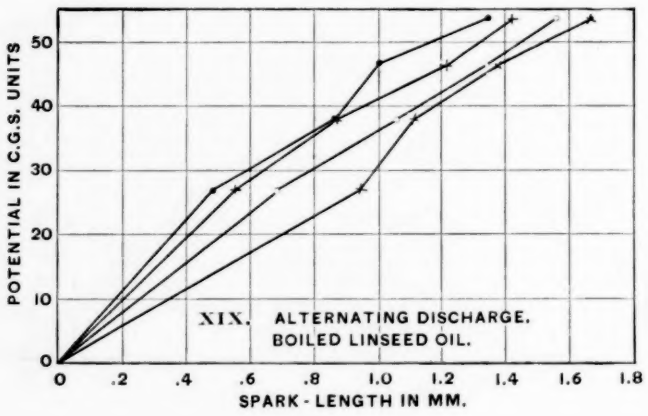

Fig. 20.

Table XXIII.

DIELECTRIC: WATER WHITE DISTILLATE.

Maximum Potential-Difference.	Spheres .5cm. diam *d*	Spheres 1cm. diam. *d*	Spheres 2cm. diam. *d*	Spheres 3cm. diam *d*
C. G. S. units.	mm.	mm.	mm.	mm.
26.98	.35	.21	.44	.66
38.01	.60	.46	.76	.86
46.55	.76	.89	1.04	1.22
53.76	.86	1.24	1.49	1.75

Table XXIV.

DIELECTRIC: XYLOL.

Maximum Potential-Difference.	Spheres. .5cm. diam. *d*	Spheres. 1cm. diam. *d*	Spheres. 2cm. diam. *d*
C. G. S. units.	mm.	mm.	mm.
26.98	.20	.27	.28
38.01	.35	.34	.43
46.55	.48	.42	.60
53.76	.57	.46	.69

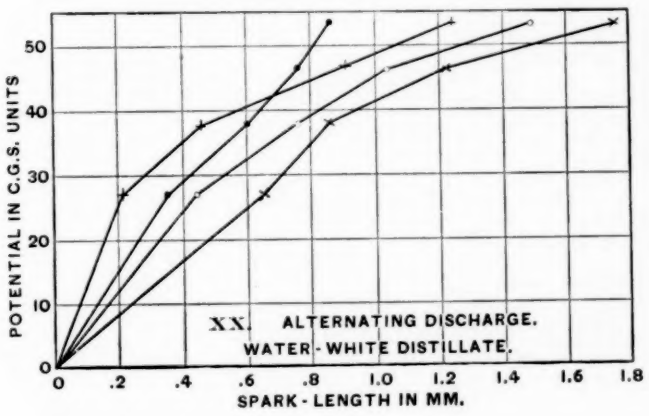

Fig. 21.

It will be seen from Tables XIX.–XXIV. that, just as in the case of the electrostatic discharge, the dielectrics used in these experiments are electrically weaker for spheres of 3 cm. diameter than for the smaller spheres of 1 cm. and 2 cm. diameter, while the smallest

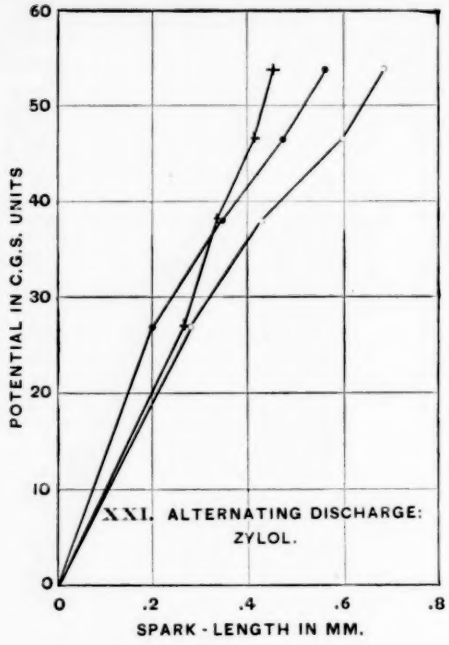

Fig. 22.

spheres exhibit the same anomalous behavior as before. The results for turpentine and linseed oil are in fair agreement with the results obtained by Steinmetz, for the values of the spark-gap for plane electrodes would be somewhat greater than those for spheres 3 cm. in diameter. The greatest difference between the results here recorded and those given by Steinmetz is in the case of air. In the last two columns of Table XIX. are given the values of d corresponding to the given differences of potential, which have been calculated from the results of Steinmetz and Siemens. It will be noticed that the values of d obtained for the largest spheres lie in every case between those obtained by Steinmetz and Siemens. This difference seems to point to some influence of the frequency of the alternations upon the spark-length, for the frequencies in the experiments of Steinmetz and Siemens and in those here recorded were 150, 100 and 125 respectively. We have already mentioned Pierce's objection on the score of the separation of the electrodes by the interposition of solid dielectrics, but it would appear to have

little weight here, because, in the experiments with spherical electrodes, no solid body had to be interposed in order to separate them. In fact, the results given in the last three columns of Table XIX. appear to bear out Jaumann's contention that the more rapidly the potential is changed the less will be the potential required to spark across any given distance. This, however, is not equally well borne out by the experiments on turpentine and linseed oil, for the values of the spark-length calculated from Steinmetz's results are only about equal to the values which might be expected for planes.

In conclusion, I would express my sincere thanks to Professor A. G. Webster for his advice and suggestions during the course of the work, and to the President and Trustees of Clark University for the facilities placed at my disposal for the prosecution of the experiments.

LABORATORY OF CLARK UNIVERSITY,
WORCESTER, MASS.

ON THE THEORY OF MAGNETO-OPTIC PHENOMENA. II.[1]

BY C. H. WIND.

§ 7. *Reflection and Refraction at the interface of two media, one of which is transparent and shows no Hall-effect, while the Coefficient of Absorption and the Hall-constant of the other possess sensible values; a Magnetic Force is applied parallel to the Plane of Incidence.*

TWO extreme cases are separately examined, namely those of polar and of equatorial reflection. The first problem solved for the two cases is the following: What must be the constitution of the incident light in order that the refracted ray may be circularly polarized as either of the rays considered in paragraph 6?

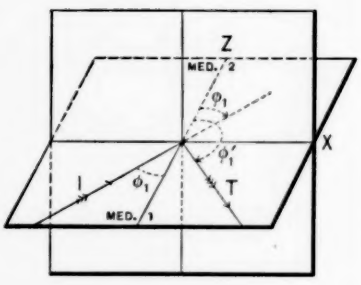

Fig. 2.

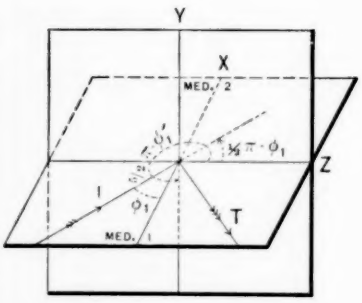

Fig. 3.

The system of axes has different positions for the two cases as shown in diagrams 2 and 3, in which I and T are the incident and

[1]Continued from p. 51.

reflected rays respectively, φ_1 is the angle of incidence, and φ_1' the supplement of the angle of reflection.

Our conditions of continuity (II),(IV), (V) and (VI) become :

POLAR.		EQUATORIAL.	
$(w)_1 = (w)_2$;	$(\alpha)_1 = (\alpha)_2$;	$(u)_1 = (u)_2$;	$(\alpha)_1 = (\alpha)_2$,
$(X)_1 = (X)_2$,	$(\beta)_1 = (\beta)_2$,	$(Y)_1 = (Y)_2$,	$(\beta)_1 = (\beta)_2$,
$(Y)_1 = (Y)_2$;	$(\gamma)_1 = (\gamma)_2$;	$(Z)_1 = (Z)_2$;	$(\gamma)_1 = (\gamma)_2$.

The incident light may be characterized by

$$\left.\begin{aligned} u &= \sigma_1 \cos\varphi_1 \,.\, \mathrm{P}_1 \\ v &= s_1 \,.\, \mathrm{P}_1 \\ w &= -\sigma_1 \sin\varphi_1 \,.\, \mathrm{P}_1 \end{aligned}\right\}, \qquad \left.\begin{aligned} u &= \sigma_1 \sin\varphi_1 \,.\, \mathrm{P}_1 \\ v &= s_1 \,.\, \mathrm{P}_1 \\ w &= -\sigma_1 \cos\varphi_1 \,.\, \mathrm{P}_1 \end{aligned}\right\},$$

where

$$\mathrm{P}_1 = e^{\delta[t - R_1(x\sin\phi_1 + z\cos\phi_1)]}, \qquad \mathrm{P}_1 = e^{\delta[t - R_1(x\cos\phi_1 + z\sin\phi_1)]},$$

$$R_1^{\,2} = \frac{4\pi\rho_1}{\delta},$$

and the corresponding expressions for the components of $\mathfrak{F}$ and $\mathfrak{H}$, the reflected light on the other hand by the same expressions with σ_1' s_1', φ_1' and P_1' instead of σ_1, s_1, φ_1 and P_1, and the refracted light by

$$\left.\begin{aligned} u &= a_2 \cos\varphi_2 \,.\, \mathrm{P}_2 \\ v &= \pm i a_2 \,.\, \mathrm{P}_2 \\ w &= -a_2 \sin\varphi_2 \,.\, \mathrm{P}_2 \end{aligned}\right\}, \qquad \left.\begin{aligned} u &= a_2 \sin\varphi_2 \,.\, \mathrm{P}_2 \\ v &= \pm i a_2 \,.\, \mathrm{P}_2 \\ w &= -a_2 \cos\varphi_2 \,.\, \mathrm{P}_2 \end{aligned}\right\},$$

where

$$\mathrm{P}_2 = e^{\delta[t - R_2(x\sin\phi_2 + z\cos\phi_2)]}, \qquad \mathrm{P}_2 = e^{\delta[t - R_2(x\cos\phi_2 + z\sin\phi_2)]},$$
$$R_2 = R_{20}(1 \mp \mu i\cos\varphi_2), \qquad R_2 = R_{20}(1 \mp \mu i\sin\varphi_2),$$

$$R_{20}^{\,2} = \frac{4\pi\rho_2}{\delta}, \; \mu = \tfrac{1}{2}\rho_2 q N,$$

and the corresponding formulæ for the components of $\mathfrak{F}$ and $\mathfrak{H}$— finally for the whole set of formulæ

$$\delta = -\frac{2\pi i}{T}.$$

The above conditions of continuity give us then, besides the relation

$$\varphi_1' = \pi - \varphi_1, \tag{63}$$

a set of formulæ, by which the constants σ_1, s_1, σ_1' and s_1' are expressed in terms of a_2. In these formulæ, which we shall not write down here, there occur some double signs $\pm$ and $\mp$ according to the two senses of circular polarization, which may be assumed for the refracted ray.

Let us now assume in the second (metallic) medium *two* circularly polarized rays propagating at the same time, one being right-handed and the other left-handed. One of these will correspond to the above equations with the upper of the double signs, and will be farther determined by the value a_{2+} for the constant a_2; the other corresponds to the same equations with the lower signs, and is farther determined by the value a_{2-} for the constant a_2. In agreement with this notation we shall distinguish the corresponding values of σ_1 etc., similarly by a suffix + or −.

On the other hand, we shall suppose the incident light to be plane polarized. Let us now direct our attention especially to the constitution of the reflected ray, leaving the refracted ray for the present out of consideration. I shall distinguish two cases, which we shall refer to as *principal cases*.

I. The plane of polarization of the incident ray is perpendicular to the plane of incidence. In this case $s_{1+} + s_{1-} = 0$ and we have to determine the value b_s of $s_1'_+ + s_1'_-$ and the value a_p of $\sigma_1'_+ + \sigma_1'_-$ when $s_{1+} + s_{1-}$ is put equal to unity.

II. The plane of polarization of the incident ray is parallel to the plane of incidence. Therefore $\sigma_{1+} + \sigma_{1-} = 0$, and the question to be answered is: What will be the value a_s of $s_{1+} + s_{1-}$ and the value b_p of $\sigma_1'_+ + \sigma_1'_-$ when $s_{1+} + s_{1-}$ is put equal to unity?

As a solution to the questions as stated we find the formulæ

POLAR.	EQUATORIAL.
$a_p = \dfrac{\bar{p}_2 \bar{p}_3}{\bar{p}_4 \bar{p}_1} \cdot e^{i(\delta_2 + \delta_3 - \delta_4 - \delta_1)}$	
$b_s = \dfrac{\sigma^4}{\bar{p}_1^2 \bar{p}_4 \cos^3\varphi_1} \cdot \dfrac{\rho}{2AV} \cdot e^{i(4\tau - 2\delta_1 - \delta_4 + \frac{1}{2}\pi + S)}$	$b_s = \dfrac{\sigma^3 \sin\varphi_1}{\rho \bar{p}_1^2 \bar{p}_4 \cos^3\varphi_1} \cdot \dfrac{\rho}{2AV} \cdot e^{i(3\tau - \omega - 2\delta_1 - \delta_4 + \frac{1}{2}\pi + S)}$

(84)

POLAR. EQUATORIAL.

$$a_s = \frac{\bar{p}_2}{\bar{p}_1} \cdot e^{i(\delta_2 - \delta_4 + \pi)}$$

$$b_p = \frac{\sigma^4}{\bar{p}_1^2 \bar{p}_4 \cos^3\varphi_1} \cdot \frac{\rho}{2\Lambda V} \cdot e^{i(4\tau - 2\delta_1 - \delta_4 + \frac{1}{2}\pi + S)} \qquad b_p = \frac{\sigma^3 \sin\varphi_1}{\rho \bar{p}_1^2 \bar{p}_4 \cos^3\varphi_1} \cdot \frac{\rho}{2\Lambda V} \cdot e^{i(3\tau - \omega - 2\delta_1 - \delta_4 - \frac{1}{2}\pi + S)} \quad (85)$$

Λ and V representing wave-length and speed of the light considered in the first (transparent) of the two media. The other magnitudes that occur here for the first time are defined by the equations

$$\rho e^{iS} = qN \qquad (82)$$

(ρe^{iS} or qN is the quantity which we shall afterwards refer to as the Hall-constant),

$$\sigma e^{i\tau} = \frac{R_{20}}{R_1}, \qquad (80)$$

and the following equations

$$\bar{\rho} e^{i\omega} = \sqrt{1 - \sigma^{-2} e^{-2i\tau} \sin^2\varphi_1},$$

$$m = \frac{\sigma_1 \rho}{\cos\varphi_1},$$

$$tg\vartheta_1 = \frac{m \sin(\tau + \omega)}{1 + m\cos(\tau + \omega)}, \qquad \bar{p}_1^2 = 1 + m^2 + 2m\cos(\tau + \omega),$$

$$tg\vartheta_2 = \frac{m \sin(\tau + \omega)}{-1 + m\cos(\tau + \omega)}, \qquad \bar{p}_2^2 = 1 + m^2 - 2m\cos(\tau + \omega),$$

$$tg\vartheta_3 = \frac{m \sin(\tau + \omega)}{-tg^2\varphi_1 + m\cos(\tau + \omega)}, \qquad \bar{p}_3^2 = tg^4\varphi_1 + m^2 - 2m\, tg^2\varphi_1 \cos(\tau + \omega),$$

$$tg\vartheta_4 = \frac{m \sin(\tau + \omega)}{tg^2\varphi_1 + m\cos(\tau + \omega)}, \qquad \bar{p}_4^2 = tg^4\varphi_1 + m^2 + 2m\, tg^2\varphi_1 \cos(\tau + \omega).$$

From formulæ (84) and (85) we see that as a consequence of the magnetization there will appear in addition to the ordinary reflected light (a_p, a_s), which, in both the principal cases, has its plane of polarization the same as that of the incident light itself and remains unaltered, another component (b_s, b_p) with a plane of polarization perpendicular to that of the ordinary reflected light. This new component is called the *magneto-optic component*. Its appearance con-

stitutes the phenomenon known as the *Kerr-effect*. If we represent the ratio of the amplitude of the magneto-optic component to the amplitude of the incident ray in the two cases by μ_p and μ_i, respectively, we find as a consequence of the above equations

POLAR.	EQUATORIAL.
$\mu_p = \mu_i = \frac{\sigma^4}{\rho_1^2 \rho_4 \cos^3 \varphi_1} \cdot \frac{\rho}{2AV}$ (90)	$\mu_p = \mu_i = \frac{\sigma^3 \sin \varphi_1}{\rho \rho_1^2 \rho_4 \cos^3 \varphi_1} \cdot \frac{\rho}{2AV}$ (90)

Denoting the phase differences of the magneto-optic component, relatively to the parallel polarized ordinary reflected ray, for the first and the second principal cases by m_p and m_i respectively, we find similarly:

	POLAR.	EQUATORIAL.	(13)
(91)	$m_p = m_i - \pi = 4\tau - \delta_1 - \delta_2 - \delta_4 - \frac{1}{2}\pi + S.$	$m_p = m_i = 3\tau - \omega - \delta_1 - \delta_2 - \delta_4 - \frac{1}{2}\pi + S.$	(91)

The quantities σ and τ depend by the equations

$$tg\, 2(\tau - H) = -\cos 2I\, tg 2H \tag{88}$$

and

$$\sigma = tgI \sqrt{\frac{\cos 2H}{\cos 2(\tau - H)}} \tag{89}$$

upon the optical constants I and H known as principal angle of incidence and principal azimuth.

The expressions (90) and (91) differ from those derived from Van Loghem's dissertation[1] as given by Sissingh[2] and Zeeman[3] for the same quantities. This difference arises on one hand from the fact that our point of departure is Maxwell's system of equations and not that of Helmholtz, as we have in this way gotten rid of a certain quantity, A, which is not completely measurable and which is found in Van Loghem's expressions for μ_i and μ_p. On the other hand, a new quantity, S, not occurring in Van Loghem's equations, has made its appearance in ours. This S is the argument of the Hall-constant, the latter being by our fundamental hypothesis assumed to be a complex quantity.

[1] Van Loghem, Doctor dissertation, Leyden, 1883.
[2] Sissingh, Arch. néerl. 27, p. 236, 1893.
[3] Zeeman, Arch. néerl. 27, p. 274, 1894.

§ 8. *Comparison between theory and experiment in the case of the Kerr-effect.*

Upon comparing the results of observations upon the Kerr-effect, by Sissingh, Zeeman and myself, with the predictions of Lorentz's theory in its original form, a discrepancy has been found[1] in the case of iron, cobalt, and nickel between the values of the phase difference observed and those computed. The difference between the observed and computed values is nearly independent of the angle of incidence and is often referred to as *Sissingh's phase difference.* This quantity S' was found to be 85°, 40°.5 and 36°.5 in the cases of iron, cobalt, and nickel, respectively. Bearing in mind the difference between Van Loghem's formulæ for m_i and m_p and ours, as characterized at the end of the last section, it is clear that these observations mentioned will completely agree with our theory, if we attribute to the argument S of our complex Hall-constant a value equal to the Sissingh phase difference S'.

The values of $\rho \cos S$ calculated from the experimentally determined ratio of amplitudes and phase difference are usually much greater than those obtained for the Hall-constant by experiments on the common Hall-effect itself. But this discrepancy does not afford any evidence against the exactness of the theory; for there is no reason whatever to suppose that the quantity $\rho \cos S$ should be the same in the case of oscillating currents of arbitrary period as in the case of steady currents.

§ 9. *Difference between the results of theory and experiment.*

The ratio of amplitudes as determined by experiment has not always been found exactly proportional to the same quantity as calculated from the theory. On the other hand, certain measurements by Zeeman with light reflected from iron and cobalt mirrors at nearly normal incidence have furnished values of S' differing by several degrees from the values previously found for other angles of incidence. The author is of the opinion that neither of these two classes of deviation is of much consequence in its bearing upon the theory. On one hand we do not know exactly the relations existing between the Hall-constant and the magnetic quantities involved, nor do we know

[1] Cf. f. i. Wind, Verh. d. Physik. Gesellsch. Berlin 13, p. 84, 1894.

everything about the magnetic properties of the metals, especially nickel, which shows the greatest difference between the observed and computed ratio of amplitudes; and there also occur large differences in the magnetic and other properties of different samples of the same metal. On the other hand, the reflecting surfaces are in most cases covered with a thin layer of foreign matter, which, perhaps, may affect to some extent the phenomenon of reflection. These facts might certainly be found sufficient to account for the discrepancies mentioned between experiment and theory.

§ 10. *Propagation of light in any medium and reflection at an interface, as in § 7; a magnetic force being applied in a direction perpendicular to the plane of incidence. A new magneto-optic phenomenon.*

Many authors have asserted that as a consequence of the symmetry of the conditions no magneto-optic effect is to be expected when the magnetic field is perpendicular to the plane of incidence; and, as a matter of fact, no such effect has hitherto been discovered. Yet upon considering this question I have not been able to see the force of the reasons which have led to the denial of any magneto-optic effect produced by such magnetization. Although it is easy to derive from principles of symmetry that a true Kerr-effect can only be caused by *parallel* magnetization, yet the same principle does not by any means exclude *any* effect whatever of perpendicular magnetization upon the reflected light. I have therefore examined what the above theory teaches in relation to this problem, by applying it to the case of perpendicular magnetization.

It appears that our theory indicates that no difference of velocity or absorption between circularly polarized rays of different sense will be caused by this kind of magnetization, nor will there be produced in either of the two principal cases a new component in the reflected light with its plane of polarization perpendicular to that of the ordinary reflected ray. But the theory also indicates that the perpendicular magnetization *should* produce an optical effect of another kind. It should affect in a certain degree the apparent values of the quantities known in the theory of ordinary metallic reflection as the *phase difference* φ and the *reëstablished azimuth* h.

The formulæ by which the alteration in these quantities can be expressed in terms of the Hall-constant and the angle of incidence (φ_1) are

$$h - h_0 = \tfrac{1}{2} \sin 2h_0 \,.\, D_1 \cos D_2, \tag{102}$$

$$\varphi - \varphi_0 = D_1 \sin D_2, \tag{103}$$

where

$$D_1 e^{iD_2} = -2\mu i^2 \frac{\sin 2\varphi_1}{\cos^2 \varphi_1 - i^2 \cos^2 \varphi_2}, \tag{101}$$

while

$$i = \sigma^{-1} e^{-i\tau}.$$

As μ can be expressed in the magneto-optic constants ρ and S, known by observations on the Kerr-effect, and other quantities known by the ordinary metallic reflection, we can make use of (102) and (103) to calculate the values of $\varphi - \varphi_0$ and $h - h_0$ for every angle of incidence.

In this manner I have computed the values given in the accompanying table as the maximum values of the effect to be expected from certain very strong magnetic fields perpendicular to the plane of incidence.

Metal	Maximal *diminution* of *phase difference*			Maximal *augmentation* of *re-established azimuth*		
	Angle of inc. being	Amount		Angle of inc. being	Amount	
		In radians:	In ¼ λ:		In radians:	In min.:
Fe	79°	0.00609	0.0039	71°	0.00582. $\frac{1}{2}$ sin $2h_0$	8.3
Co	77°	255	16	66°	109 "	1.70
Ni	75°	218	14	62°	064 "	1.03

A glance at the numbers in this table will show that even the maximum effect to be expected is in all cases very small. Only in a few cases can it fall within the reach of the common method of observation by polarizer, compensator, and analyzer. In my original paper I have therefore proposed a more sensitive method of observation, by which the effect predicted may perhaps be measured with some degree of exactness. The conclusions of this section have thus indicated a new method by which our theory may be tested.

The author having found no opportunity to undertake any experiments himself in this direction, Dr. Zeeman has kindly tried to detect the new effect on iron mirrors[1] with the apparatus of the Leyden Laboratory, mounted especially for magneto-optical work. In these experiments he employed the old method of observation, which is scarcely sensitive enough for the purpose. Yet Dr. Zeeman has found a slight variation of φ and h; moreover he has found that the effect observed is in complete agreement with theory, not only as to its direction, but even, within the limits of observation, as to its numerical value.[2]

§ 11. *The Faraday effect.*

The rotation of the plane of polarization by thin films of magnetized iron, cobalt, or nickel (discovered by Kundt) may be left out of account for the present, since the theory of such phenomena, although completely contained in our general theory, presents some additional complications. But the same effect in dielectric media, one of Faraday's grand discoveries, is very easily understood and calculated by our theory. The latter gives for the quantity commonly called Verdet's constant the value

$$\omega = \frac{n_0^3}{\Lambda^2} \cdot \frac{q}{V} \cdot 5400, \qquad (121)$$

where n_0 is the refractive index of the medium, Λ the wave-length and V the velocity of the light considered, while q is the Hall-constant (here a real quantity) per unit of magnetic force. For carbon disulphide, for example, this formula gives for q, as calculated from the known value of Verdet's constant for this medium and other data, the value :

$$q = 1.92 \times 10^{-4} \text{ C. G. S. units.}$$

§ 12. *Drude's theory of magneto-optical phenomena.*[3]

Between Drude's theory and the one here presented there is only one essential point of difference, Drude introducing into his theory

[1] Zeeman, Versl. Kon. Akad. v. Wetensch. Amst. 5, p. 183, 1896; Communications fr. th. Lab. of Physics, Leyden, No. 29.

[2] We ought to state that other theories of magnetno-optic phenomena, like Goldhammer's and Drude's, might as well as ours have led to the prediction of a magneto-optic effect of the above kind.

[3] Drude: Wied. Ann. 46, p. 353, 1892.

a real quantity as a magneto-optic constant, which should be replaced by a complex quantity in order to cause his theory to agree with ours and with experiment. Drude himself has stated in his more recent papers that his real constant is not quite sufficient to characterize magneto-optic phenomena.

For better comparing our theory with Drude's we write our fundamental equations (*A*), (*B*) and (*C*), after having eliminated $\mathfrak{C}$, in this form :

$$\left.\begin{aligned} \text{Rot } \mathfrak{F} &= -\dot{\mathfrak{H}} \\ \text{Rot } \mathfrak{H} &= 4\pi(\rho\mathfrak{F} + \rho^2 q[\mathfrak{N} \cdot \mathfrak{F}]) \end{aligned}\right\} \qquad (122)$$

Then our theory appears to depend formally on an extension of the *second* of Maxwell's equations, written in this form :

$$\left.\begin{aligned} \text{Rot } \mathfrak{F} &= -\dot{\mathfrak{H}} \\ \text{Rot } \mathfrak{H} &= 4\pi\rho\mathfrak{F} \end{aligned}\right\}, \qquad (123)$$

this extension being of no consequence at all, when the magnetic force $\mathfrak{N}$ is zero.

The original form of Lorent's theory is declared by Drudenot to be wholly satisfactory,[1] since according to his opinion there are more grounds for explaining the effect of magnetization by extending the first of the equations (123) (as was tried by that author himself), than by extending the second of those equations. As by our extension of Lorentz's theory (our hypothesis as to the constant q being complex) the general character of that theory remains quite unaltered, it may be of some importance to state here that our equations are transformed into those of Drude merely by introducing a new vector $\mathfrak{F}_D$ satisfying the equation

$$\mathfrak{F}_D = \mathfrak{F} + \rho q\,[\mathfrak{N} \cdot \mathfrak{F}], \qquad (124)$$

and eliminating the vector $\mathfrak{F}$. By doing so (123) becomes

$$\left.\begin{aligned} \text{Rot}\,(\mathfrak{F}_D - \rho q\,[\mathfrak{N} \cdot \mathfrak{F}_D]) &= -\dot{\mathfrak{H}} \\ \text{Rot } \mathfrak{H} &= 4\pi\rho\mathfrak{F}_D \end{aligned}\right\} \qquad (125)$$

and Drude's objection does not hold any longer, while we may as well write $\mathfrak{F}_D$ as $\mathfrak{F}$ in Maxwell's equations (123), these two vectors being identically the same as long as $\mathfrak{N} = 0$. Which of the two

[1] Drude, *l. c.*, p. 376, 377.

vectors $\mathfrak{F}$ or $\mathfrak{F}_D$ is in a magnetized medium considered as the electric force depends entirely upon the way in which we feel inclined to represent to our minds the mechanism of electromagnetic phenomena. In the author's original paper it is shown that to consider $\mathfrak{F}$ as the electric force corresponds to assuming a new system of material points to act upon what we call electricity in consequence of the act of magnetization; while if we consider $\mathfrak{F}_D$ as the electric force this corresponds to assuming that the act of magnetization brings about a change in the *manner* in which the system of material points, which is considered as being endowed with the electromagnetic energy, acts upon electricity.

Drude's theory is less general than ours, since it introduces Maxwell's equations in such a form as to include the hypothesis that the electric current consists of two different parts, one of which obeys Ohm's law, while the other is equal to the time variation of the electric displacement (displacement current). This special hypothesis is not introduced in our theory.

§ 13. *Goldhammer's theory.*[1]

It is not difficult to show that our fundamental equations may be readily transformed into those of Goldhammer and that in this respect our theory is equivalent to his. Yet the theory of the latter, like that of Drude, involves the special hypothesis referred to at the end of the last section.

It is true that Goldhammer denies any direct relation between the Kerr-effect and the Hall-effect,[2] but this is entirely due to the fact that he neglects the probability of the Hall-constant being largely variable with the period of vibration T.

§ 14. *Application of the principle of symmetry to reflection on a metallic mirror.*

We shall assume as evident the following principle: If any system can be considered as its own reflected image with reference to a certain plane V, not only as regards the position of the material points of the system, but also as regards the laws of their mutual

[1] Goldhammer, Wied. Ann., 46, p. 71, 1892.
[2] Goldhammer: *l. c.*, p. 96.

action, then corresponding to every possible state of motion in the system there is another possible state of motion which behaves towards the first as its reflected image with reference to the plane V. From this principle the following theorems are derived, relating to the reflection of plane polarized light either with its plane perpendicular to the plane of incidence or parallel to it (the two principal cases formerly mentioned):

I. If the mirror is not magnetized the plane of polarization of the reflected ray must bear the same relation to the plane of incidence as does that of the incident ray.

II. The same is true when *perpendicular* magnetization is applied; hence this kind of magnetization is only able to produce in the reflected ray a magneto-optic component that is polarized in the same way as the ordinary reflected ray (see paragraph 10).

III. Reflected light which is polarized in the same way as the incident ray cannot be itself modified by *parallel* magnetization; hence this kind of magnetization is able to produce merely a new component with its plane perpendicular to the ordinary reflected ray. As shown by the Kerr-effect it actually does produce such a component.

§ 15. *Applications of the law of reciprocity.*

From this law, which may be extended to magneto-optic phenomena, a fourth theorem may be readily derived.

IV. If polar or equatorial magnetization produces a magneto-optical component with its plane perpendicular to that of the ordinary reflected ray (according to III), the ratio of its amplitude to that of the incident ray must be the same in both the principal cases. In the case of polar reflection the phase differences are also the same in both the principal cases; in the case of equatorial reflection they will on the contrary show a difference equal to π.

This theorem is in perfect agreement with the formulæ previously derived from theory [see (90) and (91)].

Theorems II, III and IV enable us to predict some of the main features of the effect of magnetization on reflected light. Moreover, the direction of the reflected ray is completely determined by the peculiar periodical character of the electromagnetic vibrations,

together with the condition of continuity of the normal component of current. Therefore a special theory is wanted only in order to furnish the values of the ratio of amplitudes and of the phase difference for either of the two principal cases.

§ 16. *Interpretation of the complex nature of the Hall-constant.*

In a short note upon this subject[1] the author has already shown that the interpretation of a complex Hall-constant may be as follows: If it be assumed that the total current $\mathfrak{C}$ consists of two parts,[2] namely, a conductive current $\mathfrak{C}_1$ and a displacement current $\mathfrak{C}_2$, each of which obeys its own law as to the relation between current and electric force, we might attribute a Hall-effect to each of them; but there would be more reason to ascribe a different rotatory effect to the different parts of the current than to ascribe an equal one. We might denote the Hall-constant for the current $\mathfrak{C}_1$ provisionally by h, and for the current $\mathfrak{C}_2$ by k; we then get our equation (C) by putting

$$h = \frac{\rho}{N} \cos S + \frac{2\pi}{NT} \frac{\rho_2}{\rho_1} \rho \sin S \tag{135}$$

and

$$k = \frac{\rho}{N} \cos S - \frac{T}{2\pi N} \frac{\rho_1}{\rho_2} \rho \sin S, \tag{136}$$

and considering that $\rho\epsilon^{iS}$ stands for qN (compare equation (82)). A simple discussion of these formulæ shows that the theory which *ab initio* makes h equal to k (corresponding to Lorentz's original theory) will follow from our theory, if we assume that S is equal to zero. It shows further (see the original paper) that Drude's assumption of a real magneto-optic constant corresponds to putting the constant h equal to zero; *i. e.*, to the assumption that the conductive current in the case of optical phenomena shows *no* Hall-effect. J. J. Thomson,[3] upon comparing the Kerr-phenomenon with the Hall-effect, concludes that in the case of light vibrations no Hall-effect is to be

[1] Wind, Versl. Kon. Akad. v. Wetensch. 3, p. 82, 1894; Verhand. d. Physik. Gesellsch. Berlin 13, S. 84, 1894.

[2] Cf. the end of section 12.

[3] J. J. Thomson, Rec. Res., i. El. a. Magn., p, 486, 1893.

ascribed to the conductive current. This is in agreement with Drude's theory, but contradicts the theory here presented and also experience.

§ 17. *A physical interpretation of the fundamental equations for the Hall-effect and the Kerr-effect founded upon Lorentz's theory of the motion of electricity by ions.*

In agreement with the views of Lorentz, we shall now assume that ions exist in the ether without disturbing the contiguity of this medium. These ions may be acted upon and sometimes set into motion by forces depending upon local disturbances in the ether. As to what may be the real nature of these disturbances we do not venture to make any suggestion. We only assume that they may be described by the aid of two vectors called respectively the *dielectric displacement* $\mathfrak{d}$ and *magnetic force* $\mathfrak{H}$.

These fundamental ideas lead to a conception of electric current as the sum of the time variation of dielectric displacement in the ether and of a displacement of electricity.

Lorentz's considerations furnish at once our equations (A), (B), (I) to (VI); equation (C) is thus the only one lacking, and requires further deduction. According to Lorentz we must consider in equations (A), (B), (I) to (VI) the vector $\mathfrak{F}$ to be the force (per positive unit of charge) which, in consequence of the dielectric displacement, is acting upon an ion at rest. This vector is determined by the equation

$$\mathfrak{F} = 4\pi V^2 \mathfrak{d} \qquad (138)$$

An ion in motion is, however, acted upon not only by this force $\mathfrak{F}$, but also by an additional force, which we may call the electrodynamic force, and which is represented by the vector product of the velocity and the magnetic force $\mathfrak{H}$ at the point. This additional force is undoubtedly insignificant as long as we deal with ordinary velocities of ions, and with those magnetic intensities which result from the electromagnetic disturbances constituting light. But it acquires a sensible value when the ions are moving, a very strong magnetic field being superposed. If the magnetic force in such a field is called $\mathfrak{R}$, we have for the total force $\mathfrak{E}$ acting upon each positive unit of charge moving with the velocity $\mathfrak{v}$

$$\mathfrak{E} = \mathfrak{F} + [\mathfrak{v}.\mathfrak{R}] \qquad (139)$$

Now it is not difficult to show that equation (C) which contains the mathematical description of the Hall-effect, may be deduced from the following simple hypothesis, viz.: *The mean velocity of the positively charged ions (kations) is not exactly opposite to that of the negatively charged ions (anions).*

We have already shown that the Kerr-effect is included in the same equation (C). It is only necessary to make q a complex quantity in order to get a theory in complete accordance with observation. Now this complex nature of the Hall-constant q can be shown to follow from this new hypothesis, namely, *that the ions existing in a metallic medium are of two different classes.*

Indeed, we may imagine, I think, as existing in the metallic medium in addition to those ions to which special attention is paid in Lorentz's paper, and which I shall call *dielectric ions*, another class of ions, which I shall call by the name of *conductive ions*. These are distinguished from the first kind by the fact that in consequence of their motion they give rise to a resisting force proportional to their velocity. We shall call e the charge of an ion, and n the number of ions in unit volume, and write ε' for $\frac{1}{n}\frac{\bar{\mathfrak{v}}}{e\mathfrak{v}}$[1] in the case of conductive ions, and ε'' for $\frac{1}{n}\frac{\bar{\mathfrak{v}}}{e\mathfrak{v}}$ in the case of dielectric ions, where the horizontal line is to indicate a certain mean value of the quantity considered; again we shall call $\mathfrak{M}'$ the displacement of electricity ("elektrisches Moment") due to the motion of the first class of ions, and $\mathfrak{M}''$ that due to the motion of the second class. We thus distinguish quantities relating especially to one or the other class of ions by the single or the double accent. We then have

$$\mathfrak{E}' = \mathfrak{F} + \varepsilon' [\mathfrak{M}' . \mathfrak{R}], \qquad (141)$$

$$\mathfrak{E}' = \mathfrak{F} + \varepsilon'' [\mathfrak{M}'' . \mathfrak{R}]. \qquad (142)$$

We cannot give a rigorous deduction of the equation of motion for the separate ions. Yet by a plausible train of reasoning we are led to assume them to be of this form

[1] By a mistake the number n of ions is left out of consideration in the original (Dutch) paper.

$$\mathfrak{E}' = (\xi + \eta\partial)\,\mathfrak{M}', \tag{145}$$

$$\mathfrak{E}'' = \zeta\,\mathfrak{M}'', \tag{144}$$

where ξ, η and ζ are constant quantities relating to the medium considered.

As according to our above conception of electric current

$$\mathfrak{C} = \dot{\mathfrak{d}} + \mathfrak{M}' + \mathfrak{M}'', \tag{146}$$

we find by (145), (144), (141), (142) and 138)

$$\mathfrak{C} = p\,\mathfrak{F} + r\,[\mathfrak{N} \cdot \mathfrak{F}], \tag{C'},$$

where is put

$$\partial\left[\frac{1}{4\pi V^2} + \frac{1}{\xi + \eta\partial} + \frac{1}{\zeta}\right] = p \tag{150}$$

and

$$-\partial^2\left[\frac{\varepsilon'}{(\xi + \eta\partial)^2} + \frac{\varepsilon''}{\zeta^2}\right] = r. \tag{151}$$

By this way we see our considerations lead to the equation (C'), which is entirely equivalent to our fundamental equation (C).

Although in the original paper other conclusions will be found mentioned, I shall mention only one of them here. The considerations of the last few pages may be immediately applied to the electrical phenomena in electrolytic solutions They furnish the following expression for the Hall constant in such a medium :

$$q = \frac{1}{n[e]}\,\frac{v - u}{v + u},$$

if $[e]$ denote the ionic charge apart from sign, and u and v the "Wanderungsgeschwindigkeiten" of kations and anions.

MINOR CONTRIBUTIONS.

ON THE COEFFICIENT OF ABSOLUTE RIGIDITY OF QUARTZ FIBERS AND ITS TIME AND TEMPERATURE VARIATIONS.

BY S. J. BARNETT.

THE following investigation was undertaken at the suggestion of Professor Edw. L. Nichols, the immediate object being to find whether or not a quartz fiber would make a suitable suspension for the magnet of the great tangent galvanometer of this laboratory, mounted as a Villari torsion instrument. The results obtained, however, seem of sufficiently general interest to merit publication.

The quartz used in all the experiments except those described under II. was from a single large and perfectly clear crystal, presented by Messrs. Bonschur and Holmes, of Philadelphia, and was prepared by placing a quantity of small pieces in a deep platinum crucible and directing the flame of a small oxy-hydrogen blow-pipe downward upon the mass. The sides of the vessel and a sheath placed about them prevented the violent scattering of the heated crystal, and the surface layer was fused into an

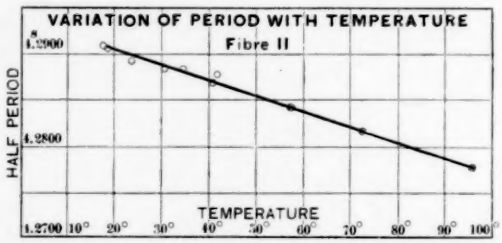

Fig. 1.

irregular disc, from which the rods, as ordinarily used, were readily formed. Rods were similarly prepared from chemically deposited SiO_2, but it is more difficult to obtain these free from air bubbles, and they were not used in the experiments. From the rods fibers were drawn by hand in the usual way, very fine fibers not being desired.

The constants investigated were the coefficient of absolute rigidity and its time and temperature coefficients—the last named involving the determination of the coefficient of linear expansion.

I. The Coefficient of Absolute Rigidity.

For these experiments the fibers were drawn with special care toward avoiding longitudinal strain and toward producing a circular cross-section. To secure as nearly a cylindrical or conical figure as possible the fibers were cut very short—even a length of three or four cms. being measured with sufficient accuracy by a fine mm. scale. To reduce errors of measurement in diameter the fibers were drawn relatively thick. This measurement was made with a high degree of accuracy with a Geneva Society micrometer microscope, 29.96 revolutions of the micrometer screw being equivalent to 0.5 mm. on a standard Rogers scale, and all the measurements being made in the center of the field. The method of torsional vibrations was used and the observations of period were made with the clock and chronograph mentioned below, every passage across the telescope wire of a vertical line upon the vibrator being recorded. The vibrator was a cylinder of brass so mounted as to have its axis continuous with that of the fiber.

With the upper end of the fiber as origin, let x be a coördinate parallel to the axis the fiber, r its radius at any point, a and b its extreme radii, and l its length. Let also n be the coefficient of absolute rigidity, T the torsional moment, and θ the angle through which the lower end of the fiber is twisted from its mean position. Then we have

$$\frac{d\theta}{dx} = \frac{2T}{\pi n} \cdot \frac{1}{r^4}, \text{ and } \theta = \frac{2T}{\pi n} \int_0^l \frac{dx}{r^4}.$$

In this expression the coefficient of T is equal to $\frac{t^2}{\pi^2 K}$, when t is the half period of vibration and K is the moment of inertia of the vibrator. If the fiber is conical, which will be nearly true if its length is small,

$$r = a + \frac{b-a}{l} x, \text{ and } \theta = \frac{2T}{\pi n} \int_0^l \frac{dx}{(a + \frac{b-a}{l} x)^4}$$

Integrating, and equating the coefficient of T to the value above given, we find, on solving for n,

$$n = \frac{2\pi K}{3t^2} \cdot \frac{l}{b-a} \cdot \left(\frac{1}{a^3} - \frac{1}{b^3} \right).^1$$

Gibson and Gregory working in the laboratory of Boys,[2] and using fibers presumably much finer than the above, found $n = 2.38 \times 10^{11}$; and Threlfall,[3] also using finer fibres, found values varying from 7.8×10^{10} to 3.4×10^{11}. These results, taken in connection with those given above, seem to indicate that the rigidity of the fibres, like the breaking weight per unit area of cross-section, increases as the diameter decreases.

[1] Franklin and Spinney, Physical Review, Vol. IV., p. 499.

[2] Boys, Journal of the Society of Arts, Sept. 27, 1889; Phil. Mag., S. 5, 30, 1890, p. 112.

[3] Threlfall, Phil. Mag., S. 5, 30, 1890, pp. 108, 109.

The mean observations and the values of n computed from this formula are given in the following table:

TABLE I.

Number of Fiber.	K	a	b	l	t	n	No. Half Vibrations Observed.
I.	0.495	0.00266	0.00272	4.30	1.348	1.41×10^{11}	96
II.	"	0.00436	0.00490	4.49	0.496	1.25×10^{11}	294
III.	"	0.00384	0.00418	3.20	0.510	1.49×10^{11}	322
IV.	"	0.00346	0.00352	3.00	0.613	1.67×10^{11}	370
V.	"	0.00389	0.00423	3.02	0.496	1.41×10^{11}	364
VI.	"	0.00387	0.00402	3.28	0.529	1.51×10^{11}	430
					Mean	1.46×10^{11}	

II. THE TIME COEFFICIENT.

The fibers were drawn by hand from rods made of small crystals fused together in the usual manner. They were each about 20 cms. long and ranged in mean radius from about 0.0028 cm. to about 0.0036 cm. They were mounted in metal tubes and attached with sealing wax to the torsion heads and to the vibrators, which were brass cylinders weighing from 8.4 to 17.2 grams. The periods were found by the method of middle elongations, ten passages of the vibrator line being usually observed for each middle elongation. Time was recorded with a chronograph and the standard Howard clock of the laboratory, the rate of which was constant. Fibers I., II. and IV. were mounted December 22, and fiber III. December 23, 1896. Nine complete sets of observations on fibers I., III. and IV., and eight complete sets on fiber II., beginning on December 31, 1896, and ending on May 10, 1897, failed to show any time effect on the periods. Thermometers inserted in all the tubes showed that the slight variations occurring were due to temperature changes. The decrease of period with increase of temperature, described in III., was first observed in these experiments. In table II. are contained the mean periods and temperatures and the limiting numbers of half vibrations in the complete sets for each fiber:

TABLE II.

No. of Fiber.	I.	II.	III.	IV.
Mean half period . .	2.1087	2.2758	2.5078	3.0166
Mean temperature . .	21°	21°	21°	21°
No. of half. vibs. in sets	660 to 1082	483 to 1019	588 to 908	497 to 757

III. The Temperature Coefficient.

If n_1 and n_2 denote the coefficients of absolute rigidity of a quartz fiber, and τ_1 and τ_2 the half periods of vibration, at temperatures t_1 and t_2; and if a_1 and a_2 denote the linear coefficients of expansion of fused quartz and brass respectively; then,[1] for a cylindrical fiber mounted with a cylindrical vibrator of brass, whose axis is continuous with that of the fiber,

$\frac{n_2}{n_1} = \left(\frac{\tau_1}{\tau_2}\right)^2 [1-(3a_1 - 2a_2)(t_2-t_1)]$ very approximately; and this must be equal to $1 + K(t_2 - t_1)$, when K is the mean temperature coefficient of the coefficient of absolute rigidity between the temperatures t_1 and t_2. On account of the variation of the viscosity of air with the temperature, this formula will not give the coefficient with extreme accuracy except in an exhausted tube, but will give a value a little too small. Considering, however, the purpose of the investigation and the errors of experiment, it was not thought worth while to take the extra trouble of making an air tight vibration tube.

All the vibrators used were cylindrical and were cut from the same rod of brass. After careful annealing, to insure an equal coefficient in every direction, the linear coefficient of expansion of the rod was measured directly by the standard method of the comparator, and from two concordant sets of observations the mean value between 18.1° and 97.9° was found to be + 0.0000182.

Threlfall and Pollock,[2] using the method of Matthiessen, obtained for the coefficient of linear expansion of fused quartz widely discordant results, their values ranging from + 0.000037 to — 0.000022, and a complete redetermination was therefore necessary. It was expected that a value intermediate between the extreme values of the coefficient for crystal quartz would be obtained. These values according to Benoit[3] are + 0.0000072 and + 0.0000133. A steam jacketed vibration tube covered with asbestos, and provided with two glass covered openings 20 cms. apart and a fine thermometer in the inner tube, was fastened vertically upon the wall and three fibers successively mounted as for torsion experiments. On account of the high upper temperature it was necessary to platinize the ends of the fibers and to solder them into the vibrator and torsion head. An upright comparator was placed in position with its microscopes 20 cms. apart focussed upon the fiber. Minute irregularities on the fibers served as good marks in the case of the first two fibers used and as thoroughly reliable marks in the case of the third fiber. One mm. (Rogers scale) was found equal to 23.16 revolutions of the

[1] The collected formulæ may be found in Threlfall, *loc. cit.*, p. 107.

[2] *loc. cit.*, p. 113.

[3] Bureau International des Poids et Mesures, Travaux et Memoires, Tome VI., p. 190.

upper micrometer head and to 20.03 revolutions of the lower. Observations were made at room and steam temperatures. With the first two fibers sometimes an extremely small negative coefficient and sometimes a like positive coefficient resulted, indicating so small an effect that it was masked by the change in length of the comparator bar, due to the variation of the room temperature during the experiments. A thermometer was therefore attached to the bar and both were wrapped with paper. The third fiber was then mounted and three sets of observations obtained, the variation of the temperature of the comparator bar being now very slight and being readily corrected for. The three sets gave each the value + 0.0000003, making this the mean linear coefficient of expansion between 23.4° and 98.8°. To see whether this extremely small coefficient was peculiar to fused quartz in the form of fibers or not, a quartz rod 20 cms. long and about 1 mm. in diameter was produced and carefully annealed in the oxyhydrogen flame. With the same comparator and tube used in the last described experiments, but mounted horizontally, the expansion of this rod was measured. A single excellent set of observations gave as mean coefficient, between 24.1° and 98.0°, + 0.0000002. This coefficient is so extremely small that it may be neglected in the reduction formula above given as affecting the results less than the experimental errors, and the formula may be used for fibers of shapes other than cylindrical.

Fibers V. and VI. were mounted in the tube already mentioned whose diameter was 2.5 cms., whlie that of the jacket was 4.5 cms. The others were mounted in a similar tube of 4.5 cms. diameter surrounded with a jacketing tube of about twice its size. The jacket contained in its lower portion, partly surrounding the inner tube, a wire coil for heating electrically, and had a projecting side tube, also near the base, for heating with a Bunsen burner. By keeping the jacket filled with water, or water and steam, a very constant temperature was obtained. The torsion heads consisted of slender brass rods fitting closely in brass tubes passing through the tops of the instruments, and in the larger tube the rod was made so long as to be adjustable in height over a range of about 18 cms. ; making it possible, from the position of the one glass covered opening, to use fibers of any length up to over 20 cms. This tube was also used in the experiments described under I. The fibres were soldered in, as in the expansion experiments. The method of observation was that described under II., and for each middle elongation ten to twenty half vibrations were usually observed. The mean observations and results for six fibers, and in part for fibre II., are given in table III. It is thus seen that the rigidity of fused quartz increases with the temperature, the converse being true of all other substances hitherto investigated. This fact also appears from the observations of Adair, but escaped the notice of the writer entirely until independently discovered in the above investigation. Adair,[1] working

[1] Threlfall, *loc. cit.*, p. 111.

TABLE III.

The fractions refer to half-sets, or middle elongations.

	Lower Temperature.				Upper Temperature.						
No. of Fiber.	No. of Sets.	Nos. of Vbs. in Sets.	Mean Temp.	Mean Period.	No. of Sets.	Nos. of Vbs. in Sets.	Mean Temp.	Mean Period.	Wt. of Vibr.	Radius of Fiber.	K
I.	1	449 to 1673	20.3	2.36018	1	543 to 1325	97.2	2.35038	32.3	——	+0.000114
III.	5	1673 to 3022	17.9	0.85011	2	1325 to 2574	96.7	0.84720	32.3	0.007	+0.000123
IV.	4	1336 to 1865	17.8	1.43155	2	1134 to 1471	96.9	1.42710	14.5	0.004	+0.000115
V.	2	1673 to 1936	19.2	1.09618	1½	1428 to 1928	98.1	1.09332	32.3	0.006	+0.000102
VI.	7½	1040 to 1929	17.6	1.21716	3½	861 to 1669	98.2	1.21307	32.3	0.006	+0.000119
VII.	3	483 to 743	18.3	2.76380	1	419	96.9	2.75506	11.9	——	+0.000117
II.	See Table IV								14.5	0.002	+0.000115
									Mean value of K =		+0.000115

under Threlfall, with a single fiber in an exhausted tube, found $K = +0.00013$, but the method of observation used did not permit of anything like the accuracy obtained in the results given above, and no attention was called to the peculiar sign of the coefficient.

The existence of another positive temperature coefficient of elasticity, in the case of rubber, has been claimed by several investigators. Schmulewitsch,[1] by observing the increase with temperature of the frequency of the lateral vibration of a stretched string; Graetz,[2] by the method of torsional vibrations; and Exner,[3] by measuring the velocity of sound in rubber at different temperatures; have all obtained positive value. But these methods, in the case of rubber, are all open to fatal objections;[4] and Russner[5] has finally shown, by direct measurements of the

[1] Pogg. Ann., Vol. 144, 1871, p. 280.

[2] Wied. Ann., Vol. 43, 1891, p. 354.

[3] Wiener Ber. 69 II., 1874, p. 102.

[4] See A. Winkelmann, in Winkelmann's Handbuch der Physik, Vol. 2, II., pp. 65, 66; also Russner, Wied. Ann., Vol. 43, 1891, p. 533.

[5] Russner, loc. cit.

stretch modulus at different temperatures, that the coefficient decreases with increase of temperature. In the case of fused quartz, however, the objections to the method of torsion break down. Since the coefficient of expansion appears to be practically the same for a weighted fiber and an annealed rod, and this indicates that no such elastic æalotropy exists in this substance as Kundt[1] has shown to exist in rubber.

In Table IV. are given the observations upon fiber II. at a number of different temperatures, and the relation between temperature and period is exhibited graphically in the accompanying chart (Fig. 1). The relation is seen to be a linear one.

TABLE IV.

Order of Experiment.	No. of Half Vibrations.	Interval bet. the Mid. Elongations.	Temperature.	Half Period.	Remarks.
		s	°	s	
1	564	2419.06	23.4	4.2891	
2	250	1069.41	96.0	4.2776	
3	492	2111.07	17.8	4.2908	
4	318	1363.64	34.3	4.2881	
5	167	716.03	41.5	4.287[6]	Experiment 5 very poor—worthless beyond fourth figure.
6	177	758.30	57.3	4.2842	
7	212	907.73	72.5	4.2817	
8	175	750.45	30.1	4.2883	
9	443	1900.63	18.8	4.2904	
10	420	1796.24	40.3	4.2869	

PHYSICAL LABORATORY OF CORNELL UNIVERSITY.

ON THE SPECIFIC INDUCTIVE CAPACITY OF CERTAIN OILS.

BY ARTHUR L. CLARK.

THE use of oils as insulators has now become so frequent that a knowledge of their various electrical properties has become desirable. The dielectric strength of various oils has been investigated by Mr. T. W. Edmondson, in a paper in the present number of the PHYSICAL REVIEW. It was suggested to me by Professor A. G. Webster that the dielectric constant or specific inductive capacity of these liquids might be conveniently studied by Quincke's bubble method. Knowledge of these two properties will enable us to decide between different oils in constructing condensers.

[1] Kundt, Pogg. Ann., Vol. 141, 1874, p. 125.

By blowing a bubble of air between two condenser plates immersed in a liquid dielectric, Quincke[1] was able to measure the pressure perpendicular to the lines of force, and by means of the expression

$$P = \frac{K - 1}{8\pi} \frac{F^2}{d^2}$$

he determined the specific inductive capacity of the medium.

P in the above is the pressure per unit of area, K the dielectric constant, F the difference of potential between the plates, and d their distance apart.

When the plates are electrified the bubble is contracted by the pressure and the change of pressure on the air in the bubble is detected by a manometer which communicates with it.

The writer has repeated the experiments with apparatus substantially the same as Quincke's to ascertain whether the method would be useful in determining the dielectric constant of oils. Two brass condenser plates about 15 cm. in diameter and 4 mm. thick, carefully ground, are separated by three glass pillars about 4 mm. high and immersed in the liquid, resting on three small brass knobs cemented to the bottom of the dish, which in turn rests on a small stand provided with levelling screws. Through the upper plate is a hole, about 1 mm. in diameter. This communicates with a glass tube rising about 10 cm. to a T; one end of the T opens into a tube filled with calcium chloride and provided with a cock, the other end is connected with the manometer containing kerosene of density 0.782. Air is blown through the calcium chloride tube forming a bubble from 3 to 5 cm. in diameter, the cock is closed and the pressure read from the manometer with a micrometer microscope. When the plates are electrified, the bubble contracts and the manometer shows an increase in pressure, which is read. The difference in the two readings, multiplied by g and by the density of the manometer liquid, gives P. The difference in potential is read at the same time from the electrometer; the electrometer telescope and the micrometer microscope being arranged so as to allow simultaneous readings, one eye to each instrument.

The electrometer is the simplest kind of quadrant electrometer. It consists of a brass needle connected to earth, suspended by a phosphor-bronze wire over two quadrants connected together and with one of the condenser plates, the other being connected to earth. The needle is damped with a mica vane dipping into glycerine. The instrument, after being adjusted, was calibrated by an absolute electrometer and throughout the range of potentials required the calibration curve was found to be very approximately a true parabola.

[1] Quincke, "Electrische Untersuchungen," Wied. Ann. XIX., p. 705, 1883.

The apparatus is excited by a Holtz machine turned by a small electric motor which is governed by a variable carbon resistance in the armature circuit, giving any required velocity. The regulator is simply a few plates of carbon under pressure. This form of regulator was found very convenient because the speed necessary varies very much with the dielectric used and also with the weather.

The method of operating is as follows: The micrometer cross hair is put at any desired deflection of the manometer and then by carefully regulating the speed the potential difference is slowly increased until the pressure on the bubble reaches the desired amount, when the potential is read from the electrometer. A good condenser, for steadying the potential, is an essential part of the apparatus. The one used consisted of four micanite plates coated with tinfoil. Each had a capacity of about 10,000 cm.

Observations may be made very rapidly. The apparatus may be set up and the experiment made in thirty minutes.

Various commercial oils were examined with fair results. Ordinary kerosene insulated best of all, while the specimens of linseed and sperm oils and turpentine tried gave no results. Olive oil insulated so poorly that only a very few readings could be taken.

From the formula given above,

$$K - 1 = \frac{8\pi d^2 P}{F^2},$$

and $P = \rho g h$, where ρ is the tensity of the manometer liquid, g the acceleration due to gravity, and h the difference in height of the two arms of the manometer.

Then

$$K - 1 = \frac{8\pi d^2 g \rho h}{F^2}$$

or, if we put

$$\frac{F^2}{h} = q$$

$$K - 1 = \frac{8\pi d^2 g \rho}{q}.$$

The value of the constant numerator is 2699. The experiment determines q.

The following tables give values of h, F and q, from which are deduced the values of K:

ORDINARY KEROSENE.

h	F	q
.0784	13.2	2220
.0896	14.1	2220
.1010	15.0	2230
.1120	15.7	2200
.1230	16.5	2220
.1340	17.1	2170
		Mean 2210

$K = 2.22$

WATER WHITE. 150°

h	F	q
.056	11.0	2160
672	12.1	2170
.784	13.1	2180
896	14.1	2220
.101	14.9	2200
112	15.7	2200
		Mean 2188

$K = 2.23$

EXPORT DISTILLATE.

h	F	q
.0672	12.7	2400
.0784	14.0	2500
.0896	15.2	2570
.1010	16.1	2540
.1120	17.0	2570
		Mean 2516

$K = 2.06$

XYLOL.

h	F	q
.0560	10.6	2000
.0672	11.8	2070
.0784	12.7	2060
.0896	13.5	2030
.1010	14.2	2000
.1120	14.9	1980
.1230	15.6	1980
		Mean 2017

$K = 2.34$

BENZOL.

h	F	q
.0560	10.5	1970
.0672	11.5	1970
.0784	12.4	1940
.0896	13.2	1940
.1010	14.0	1940
.1120	14.7	1930
		Mean 1948

$K' = 2.38$

PARAFFIN OIL.

h	F	q
.0560	9.2	1510
.0672	10.2	1530
.0784	11.2	1600
.0896	12.0	1610
.1010	12.8	1620
.1120	13.5	1620
		Mean 1581

$K' = 2.71$

WHITE CASTOR OIL.

h	F	q
.0560	6.3	720
.0672	6.8	680
.0784	7.3	680
.0896	7.8	680
.1010	8.2	670
.1120	8.8	690
		Mean 687

$K' = 4.92$

NEATSFOOT OIL.

h	F	q
.0560	8.2	1200
.0672	9.1	1240
.0784	9.6	1180
.0896	10.4	1210
.1010	11.2	1220
.1120	11.7	1220
.1230	12.4	1250
		Mean 1217

$K' = 3.22$

COTTONSEED OIL.

h	F	q
.0560	7.2	930
.0672	7.9	930
.0784	8.5	930
.0896	9.2	950
.1010	9.8	940
.1120	10.3	950
		Mean 938

$K = 3.88$

SWEET NEUTRAL OIL.

h	F	q
.0560	10.5	1970
.0672	11.5	1970
.0784	12.5	1990
.0896	13.5	1980
.1010	14.2	1980
.1120	15.1	2030
		Mean 1986

$K = 2.36$

HIGH GRAVITY OIL.

h	F	q
.0560	11.6	2400
.0672	12.7	2400
.0784	13.7	2400
.0896	14.7	2410
		Mean 2402

$K = 2.12$

OLIVE OIL.

h	F	q
.0112	3.46	1070
.0224	4.90	1070
.0336	6.00	1070
.0448	6.90	1070
		Mean 1070

$K = 3.52$

CLARK UNIVERSITY, WORCESTER, MASS, July, 1897.

NEW BOOKS.

The Induction Coil in Practical Work including Röntgen X-Rays. By LEWIS WRIGHT. 8vo, pp. vi+172. London, Macmillan & Co., 1897.

Seldom has an instrument of purely scientific value met so sudden a popular demand as the Induction coil, which, by the publication of Röntgen's discovery, was given the added importance attaching to a piece of commercial apparatus. But a few months ago to be found only in the laboratories, induction coils now form part of the equipment of hospitals, physicians, and of many private individuals. The consequence is that this delicately constructed apparatus finds itself in the hands of many unacquainted with its proper use and care. Its literature, moreover, naturally partaking of the character of the subject, is purely scientific, and he who is untrained in physical science finds himself in a labyrinth of technicalities from which no guide can be found to lead him out.

To meet the popular demand for simple instruction upon coils, Mr. Wright has given us "The Induction Coil in Practical Work." "It is written simply and solely as a practical help to the efficient and safe use of an Inductive Coil." The title of the book does not imply all that is to be found between its covers, for of the eight chapters six are devoted to other accessories to X-ray equipment than the coil itself. Chapter I treats of Electrical Induction; Chapters II and III of Coils; Chapter IV of Experiments with Coils; Chapter V of Discharge in Partial Vacua; VI of Spectrum Work; VII of Discharge in High Vacua; and Chapter VIII treats of Röntgen Rays.

The attempt is made in Chapter I., under the title Electrical Induction, to compass within the limits of 22 pages a large part of what is included in the more elementary texts on Electricity. Though the author preserves his usually clear style, it would seem that this attempt is out of place as being too simple for one unacquainted with the subject and too much abridged for the beginner. The chapter on coils answers the purpose intended admirably. A very clear description—with excellent diagrams—of the construction of a coil is given, and reliable instructions are laid down for discovering and remedying its disorders. This part of the book alone would justify its appearance. The chapter on Miscellaneous Experiments is relevant as showing the nature of the spark discharge, while that on the Discharge in Partial Vacua is, as any brief treatment of

this subject must be, largely a reproduction of a part of Crooke's original paper upon this subject. The chapter on Spectrum Work, it would seem, would have no especial interest to those for whom the book is intended and but very little relevance to the subject in hand—Induction Coils.

In the chapter on X-rays the author gets nearer his subject, for these have come recently to be considered the prime product of the induction coil ; and he who purchases a guide to the use of the coil may reasonably expect to find considerable space devoted to this subject. After discussing the nature of the rays and their relation to Lenard and ultra-violet radiations, the book gives valuable suggestions regarding the choice of tubes for different sizes and kinds of coils, for penetration, definition, etc. Some useful information regarding fluorescent screens is given, together with formulas for their preparation.

That part of the work devoted to coils, tubes, and screens can not fail to prove of interest and value to those for whom it is intended, while the other parts will furnish material for those same individuals to attack as they become more interested in the subject. The book is, on the whole, well suited to its purpose. It is attractive in appearance, both in its interior and exterior ; the paper is firm, the type is large and clear ; while the diagrams are unusually instructive. Although it is lacking an index, its good paragraphing and its method of introducing new subjects by double-faced type, render it easy to find whatever is wanted.

HENRY E. LAWRENCE.

Some Unrecognized Laws of Nature. By IGNATIUS SINGER and LOUIS BERENS. 8vo, pp. xvi+511. New York, D. Appleton & Co., 1897. London, John Murray.

It is a long time since so thoroughly bad a piece of work in the domain of physics as this has come to our notice. So numerous are the blunders of these would-be epoch-makers, and so extraordinary is their failure to grasp the significance and to appreciate the value of the cumulative evidence upon which the established principles of the science are based, that no physicist is likely to be deluded into reading beyond the first few pages of their book. He has not to read far before he finds *force* and *energy* used as synonymous and interchangeable terms, and used in a way to indicate that the authors do not comprehend the accepted significance of either. He is then told (page 38) that *two quantities of matter which are equivalent as regards the attraction of the earth do not have the same relative weights in vacuo when compared at different altitudes, or different temperatures*, and (p. 41) that *pendulums of the same length and weight, but consisting of different substances, which swing together in one place would not swing together in every part of the globe.* Should the reader's interest survive these and

the numerous other absurdities contained in the opening pages of the volume, or should he persist in his reading in the hope of discovering in what manner the authors propose to establish such startling propositions as those just cited, a glance at their discussion of the laws of motion (p. 48 et seq.) would suffice to convince him that they were not qualified to prove or disprove any proposition whatever.

To proceed further with the serious reading of this book is an impossibility to any scientific mind, but it is only necessary to open casually here and there in order to discover that the entire work is a mass of absurdities. They are masked by a show of erudition which may, perhaps, deceive those who are innocent of all knowledge of science, but which will only heighten the disgust of every one who is acquainted even with the rudiments of physics.

The volume is one which might be passed by in silence but for the fact that it is put forth under the auspices of well-known publishers on both sides of the Atlantic. It will for that reason find many readers who are not able to discriminate at a glance between the pretentions of the writers and real knowledge of the subject with which they deal. To such readers the perusal of a book like this is worse than a waste of time. The placing of it in their hands is an imposition and a wrong.

E. L. N.